8. Image Analysis of Sound Pressure

This subsection contains a description of the image analysis technique. Image analysis provides a solution of the wave equations that can be interpreted in terms of "rays." That is, sound acts as if it propagates along straight paths and bounces off walls like billiard balls. A ray analysis of this kind is called "geometrical optics." It is shown in a separate geometrical optics section that an approximate solution of the wave equations for high frequencies generally leads to ray behavior. For some geometries, such as a rectangular room, the behavior holds at even very low frequencies.

Image Solution for Reflection From an Infinite Flat Wall

Reflection from a flat wall is one example of a "boundary-value" problem. A exact rigorous solution to a sound boundary value problem must satisfy two conditions: (1) it must be a valid solution of the sound wave equations; and (2) it must satisfy the boundary conditions at the reflecting surface. Assuming a rigid wall, the boundary condition is that the mean molecular velocity component normal to the wall is zero.

An image solution for the infinite flat wall reflection problem is the sum of two sound waves: the original wave, and a wave emanating from a hypothetical image on the other side of the wall. The amplitude and phase of the image source is equal to the amplitude and phase of the original source. It is exactly like an image that would be seen if the wall were a mirror. At the wall, the pressure from the two waves adds, and the normal velocity components totally cancel. It is a rigorous solution to the boundary value problem at any frequency.

If the wall is not perfectly rigid, the normal velocity component is not quite zero. The image solution is altered by reducing the image amplitude by a wall reflection coefficient, to match the new boundary condition. The reflection coefficient is assumed to be independent of frequency and angle. A different coefficient can be used for each surface.

Image Solution for a Room

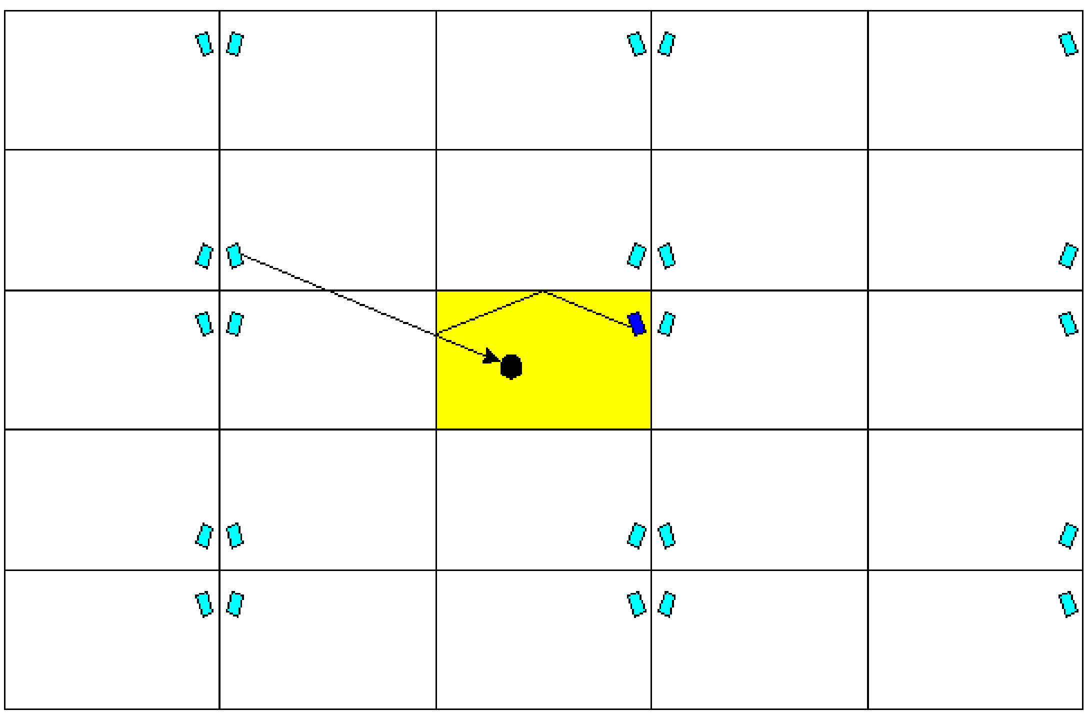
We are of course interested in rooms, not infinite walls. But the same image idea still works, it just takes an infinite number of images! This is not as bad as it sounds, as described below. The main point is that this is a perfectly valid solution, even when the wavelength is very large. This technique has been applied previously by Allen and Berkley, who also show that the image solution is mathematically equivalent to modal analysis, for rigid walls. For non-rigid walls they point out that the image solution may no longer be exact, but can still be a fairly good approximation.

The primary limitation of image analysis is that it is pretty unwieldy for any room geometry other than rectangular. However, a lot can be learned from the behavior of rectangular rooms. The accuracy of the image solution is mainly limited by imperfect knowledge of the wall reflection coefficients. Of course in the real world there is also stuff inside the room, and image theory is quite impractical for dealing with the effects of furniture. However image analysis provides a good engineering solution, and fairly realistic results can be obtained even with 1990's vintage home computers. Computed results are compared with measured data in the section on room acoustics.

A drawing of reflecting "rays" from a loudspeaker in the section on my real-world music room illustrates the formation of the image. When loudspeaker measurements are made in a room, the image sound contribution is seen in the measured data exactly as if the image physically exists; an example of this is discussed in the section on wall treatments in my music room, and further discussion is included in the section on room acoustics.

Loudspeaker Images in a Rectangular Room

For a rectangular room, the images proliferate exactly like a hall of mirrors, in 3-dimensions. In the figure below, the first few images in 2-dimensions are shown. The yellow rectangle represents the real room. The small dark blue rectangle represents a loudspeaker, and the black dot a listening position. The other rectangles are image rooms, each containing a small light blue image loudspeaker. In the image theory representation, all of these image speakers radiate a spherical wave towards the listening position. The amplitude is inversely proportional to the square of the distance from the image to the listening position, and the phase is proportional to the distance. The amplitude must also be adjusted to account for reflection losses. The total sound at the listening position is the coherent sum of all of the image contributions. Each of the images mimics a real reflection path from the real loudspeaker. One such path is illustrated in the figure. The real path goes from the speaker to the wall at the top of the rectangle, reflects back to the wall on the left, and finally arrives at the listening position. The path from the image loudspeaker is exactly the same length, and the number of surfaces it crosses equals the number of real reflections.



Since the strength of an image is reduced by each reflection the number of reflections can be limited to 10 or so. (With a typical room geometry, and 10 reflections there are over 1,500 images in 3-dimensions, but this is still a lot better than an infinite number). The number of images at a given distance is roughly proportional to the square of the distance. The voltages tend to combine incoherently, which just balances the 1/r amplitude behavior. So without reflection loss the number of reflections making a significant contribution would be infinite. The images act exactly like additional loudspeakers and contribute to "room gain."

It really is not that difficult to write a computer program to generate the images and add up all the contributions. I did just that. Most of the results are shown in the section on room acoustics. Here only a reverberation time calculation will be presented.

Reverberation Time

The absorption coefficient α is related to the reverberation time of the room, defined as the time in seconds required for sound pressure to decay 60 dB. A standard equation for reverberation time is given by [Handbook for Sound Engineers, Sect. 5.3.4]

$$(56) \qquad RT_{60} = \frac{.049V}{-S\ln(1-\alpha)} \qquad [s]$$

where V is the room volume in ft^3, and S is the surface area in ft^2. Using the image model, it is possible to directly calculate RT_{60} for a given α . The results of using equation (55) vs. the direct calculation are shown in the table below.

An important use of this calculation is that RT_{60} can be measured, and from this, a value of α can be deduced, to improve the accuracy of the image calculation. (Table revised 3/19/98 to reflect the as-built dimensions of my music room, 7.9 feet high, 13.7 feet wide, 18.85 feet effective length, and an improved image analysis calculation which changed the results a bit).

α	Equation	Image Analysis
.05	1.93	1.51
.10	.94	.94
.15	.61	.68
.20	.44	.54
.25	.34	.45
.30	.28	.38
.35	.23	.33
.40	.19	.30

This completes my version of the Physics of Sound. I really love this stuff, and hope that I have conveyed some of the enjoyment I get from probing as deeply as I can into a subject like this. (In fact this turned out to be the completion of phase I, and lots of other subjects have been added since!)

To the list of Physics of Sound subsections.