THE MODIFIED HOPKINS-STRYKER EQUATION

by

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Abstract

From the original work of Hopkins-Stryker in 1948, through its development by Beranek in 1949, and its use by Davis in 1968 and Boner in 1969 in acoustic gain calculations, the Hopkins-Stryker equation has proven highly useful to a myriad of users. In recent years this versatile tool has been modified in the light of measurements by Peutz and Davis to account for multiple sources, semi-reverberant spaces, modifiers of critical distance, and various electroacoustic modifiers of the ratio of direct-to-reverberant sound. This paper is a thorough discussion of these modifications and their proper application in acoustic calculations.

Introduction

The Hopkins-Stryker equation and its derivatives are based on the same assumptions used by Sabine in his classical study of reverberation. Sabine predicted a stochastic process in an ergodic enclosure (i.e., randomly mixing, homogeneous space). It is this description that qualifies a "reverberant sound field" as an entity distinct from a discrete reflection or a "limited train" of discrete reflections.

It was in this context that the equation called "Hopkins-Stryker" came into being with separate terms to account for the direct sound field and the reverberant sound field.

Measurements and empirical calculations have provided a further term, when required, for semi-reverberant situations. With these "cavets" let's proceed to the equation and its variations.

Modifying the Hopkins-Stryker Equation

I. BASIC EQUATION

$$L_T = L_W + 10 \text{ LOG} \left(\frac{Q(Me)}{4\pi (D_X)^2} + \frac{4N}{S\bar{a}\ (Ma)} \right) + 10.5$$

Use when $\Delta dB \leqq 1.0$ or less

II. DIRECT SOUND LEVEL

$$L_D = L_W + 10 \text{ LOG} \left(\frac{Q(Me)}{4\pi (D_X)^2} \right) + 10.5$$

Use when ΔdB is $\geqq$ than 5.0

III. REVERBERANT SOUND LEVEL

$$L_R = L_W + 10 \text{ LOG} \left(\frac{4N}{S\bar{a}\ (Ma)} \right) + 10.5$$

Use when $\Delta dB << 0.5$

IV. ACTUAL SOUND LEVEL

$$L_{act} = L_W + 10 \text{ LOG} \left(\frac{Q}{4\pi (D_C)^2} \right) + \left(0.734 \left(\frac{\sqrt{V}}{h \cdot RT_{60}} \right) \left(\text{LOG} \frac{D_C}{D_X > D_C} \right) \right) + 10.5$$

Use when ΔdB falls between 1.0 and 5.0 dB

Where: L_T is the total sound pressure level in dB at D_X (ref. 20 upa)

L_D is the direct sound pressure level in dB at D_X (ref. 20 upa)

L_R is the reverberant sound pressure level in dB at D_X (ref. 20 upa)

L_{act} is the actual total sound pressure level in dB that occurs in semi-reverberant sound fields at $D_X > D_C$ (ref. 20 upa)

L_W is the sound power level in dB for the device providing L_D at D_X (ref. 10^{-12} watt)

ΔdB is the number of dB L_{act} is below the L_T predicted by the basic Hopkins-Stryker equation at $2D_C$

$$\Delta dB = 0.221^*\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)$$

*Metric (S.I.) 0.4
($\Delta dB > 6$ dB = 6 dB)

Q is the directivity factor (dimensionless) for the device providing L_D at D_X

D_X is the distance in feet from the source to where L_X is established

Me is any electroacoustic modifier that changes L_D but not L_R (*i.e.*, a shorter D_2)

N is the total acoustic power radiated by the system divided by the acoustic power radiated by the device or devices producing L_D at D_X

$S\bar{a}$ is the total absorption in ft^2 (Sabins)

Ma is the architectural modifier $Ma = \left(\frac{1-\bar{a}}{1-a_c}\right)$

V is the internal volume of the enclosed space in ft^3

h is the height of the ceiling in ft

RT_{60} is the '*apparent*' reverberation time in secs. for 60 dB of decay

D_C is the critical distance (*i.e.*, distance at which the Hopkins-Stryker equation makes $L_D = L_R$) in ft

$$D_C = 0.141\sqrt{\frac{Q\,S\bar{a}\,(Me)(Ma)}{N}} = 0.03121^*\sqrt{\frac{QV\,(Ma)(Me)}{RT_{60}(N)}}$$

*Metric (S.I.) 0.057

0.734** a constant obtained by multiplying 0.221 by 3.322 (See *Sound System Engineering*, page 23)

**Metric (S.I.) 1.329

This constant allows calculation of the "LOG Multiplier"

$$\text{LOG Mult.} = (3.322)\,\Delta dB = 0.734\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)$$

Notes on Above Data

I. Asymptotic limit for D_X in Hopkins-Stryker equation is:

$$10 \text{ LOG} \left(\frac{4N}{S\bar{a}\ (Ma)}\right)$$

This is significant only when $D_X >>> D_C$ (*i.e.*, $D_X = 10\ D_C$)

II. English system sabins are ft^2. Metric (S.I.) sabins are M^2.

III. Derivation of the constant "10.5" in the Hopkins-Stryker equation when English system dimensions are employed.

$$\frac{0.282ft}{1} \cdot \frac{12 \text{ in}}{1ft} \cdot \frac{2.54cm}{1 \text{ in}} \cdot \frac{1M}{100 \text{ cm}} = 0.08595M$$

$$20 \text{ LOG} \left(\frac{0.282M}{0.08595M}\right) = 10.3 \text{ dB}$$

Temperature and barometric pressure correction factor:

$$dB_{corr.} = -10 \text{ LOG} \left(\frac{\sqrt{^{o}F + 460}}{527} \left(\frac{30}{B}\right)\right)$$

Where: B is the barometric pressure in inches of HG

^{o}F is the temperature in degrees Fahrenheit

$67^{o}F$ and 30" of HG result in a correction factor of zero decibels

The additional 0.2 dB in the constant "10.5" allows for variation in standard temperature and pressure (STP)

IV. Originally as defined, L_W allowed:

One acoustic watt from a source with a Q = 1.0 to produce a sound pressure level at 0.282 ft. (0.08595 M) of 130 dB (ref. 10^{-13} watt)

The current L_W allows:

One acoustic watt from a source with a Q = 1.0 to produce a sound pressure level at 0.282 M (0.925 ft) of 120 dB (ref. 10^{-12} watt)

Thus, the ref. power was raised and the ref. dist. was increased.

$$10\ \mathrm{LOG}\left(\frac{10^{-12}W}{10^{-13}W}\right) = 10\ \mathrm{dB} \qquad 20\ \mathrm{LOG}\left(\frac{0.282M}{0.08595M}\right) = 10.3\ \mathrm{dB}$$

V. Further qualification of the Ma factor

$$Ma = \left(\frac{1-\bar{a}}{1-a_C}\right)\left(\frac{Q_{act}}{Q_{theor}}\right)$$

Where: Q_{act} is the *measured* Q .

Q_{theor} is the theoretical Q that the C_L suggests (see *Sound System Engineering*, page 44)

$\bar{a}$ is the average absorption coefficient in the space

a_C is the absorption coefficient of the area where the first reflection occurs. $a_C > \bar{a}$ must occur.

VI. Delta levels (ΔD_X)

If L_W is removed from the equations, they become ΔD_X equations yielding *relative* levels.

$$\Delta D_X = 20\ \mathrm{LOG}\left(\frac{0.282}{D_X < D_C}\right) + 10\ \mathrm{LOG}\ Q$$

Remembering that:

$$10\ \mathrm{LOG}\left(\frac{1.0}{4\pi(0.282)^2} + \frac{4}{S\bar{a}}\right) \simeq 0$$

VII. Inverse functions

When $D_X < D_C$

$$D_X = \sqrt{\frac{Q}{4\pi\left(10^{\left(\frac{\Delta D_X}{10}\right)}\right)}}$$

When $D_X \geqq D_C$

$$D_X = \left(10^{\left(\frac{\Delta D_X}{0.734\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)}\right)}\right) D_C$$

Basic Equation

$$D = \sqrt{\frac{Q}{4\pi\left(10^{\left(\frac{\Delta D_X}{10}\right)} - \frac{4}{S\bar{a}}\right)}}$$

VIII. Other useful variations

$$S\bar{a} = \frac{(D_C)^2\ N}{0.019881\ Q\ (Ma)\ (Me)} \qquad Q = \frac{(D_C)^2\ N}{0.019881\ S\bar{a}\ (Ma)\ (Me)}$$

Also see *Sound System Engineering*, Appendix VIII, pages 261-270

When a D_X value of 0.282 is used in the Hopkins-Stryker equation (Q = 1), it yields one square unit of area for the surface of a sphere of that radius in whatever dimension D_X is expressed. If D_X is in feet, then the area becomes 1 ft^2. If D_X is in meters, then the area becomes 1 M^2 (0.282 ft = 0.08595m.) (0.282m = 0.925 ft.)

$$20 \text{ LOG} \left(\frac{0.282}{0.08595}\right) = 10.3 \text{ dB}$$

When the old standard reference for L_W was 10^{-13} watt, one watt was an L_W = 130 dB.

The new standard reference for L_W is 10^{-12} watt (one picowatt), and one watt is an L_W = 120 dB.

Since, in both cases, L_W is a given *fact* when available, no power adjustment of the level ΔD_X is required. What is required is an adjustment in ΔD_X for the dimensional units ft or M. If meters are used, it is correct as written plus 0.2 dB for L_W referenced to 10^{-12} watt. If ft are used, then 10.5 dB must be added to the equation as written because the distance is shorter. (10.3 + 0.2) = 10.5 dB.

When used to obtain ΔD_X numbers without L_W, no correction is required as the ΔD_X numbers are *relative* numbers. They become absolute levels only when used with an L_W.

Peutz Modification of Hopkins-Stryker Equation

I. For D_Xs under *apparent* D_C

$$\Delta D_X = 10 \text{ LOG} \left(\frac{Q}{4\pi(D_X)^2}\right)$$

II. For D_Xs equal to or greater than *apparent* D_C

$$\Delta D_X = 10 \text{ LOG} \left(\frac{Q}{4\pi(D_C)^2}\right) + 0.734 \left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right) \left(\text{LOG}\left(\frac{D_C}{D_X \geq D_C}\right)\right)$$

III. Inverse calculations

When $D_X < D_C$

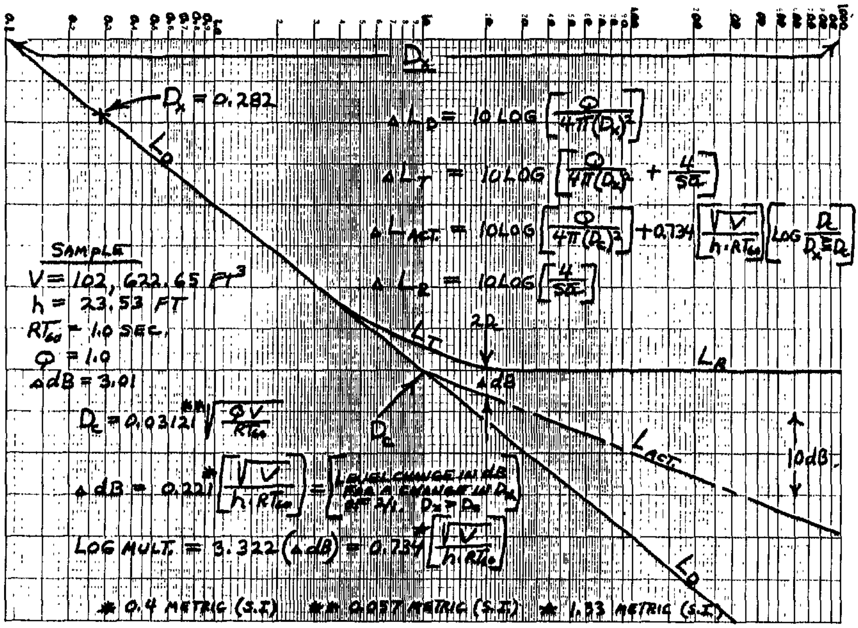
$$D_X = \sqrt{\frac{Q}{4\pi\left(10^{\left(\frac{D_X}{10}\right)}\right)}}$$

When $D_X \geq D_C$

$$D_X = \left(10^{\left(\frac{\Delta D_X}{0.734\left(\frac{\sqrt{V}}{h \cdot RT_{60}}\right)}\right)}\right) D_C$$

Notes: Ref dist for $20 \, LOG\left(\frac{ref}{D_X}\right)$ is 0.282

Apparent $D_C = 0.03121^* \sqrt{\frac{QV}{RT_{60}}}$ *0.057 metric



Acoustic Level VS Distance

Describing Q More Accurately

The measurement of the directivity factor (Q) *is always at a point*. There can be a series of points within an area that have the same Q thus allowing the concept of an "average of Qs" within an area. It is a normal practice to measure Q on axes (the zero angle axis usually being the highest output as well). Let's call this measurement Q_{axis}.

The value Q is both frequency dependent, $Q_{axis(f)}$, and, for real life devices, angularly dependent. Q_{axis} specifies the angle relative to the transducer. For angles other than the "on axis" position we could specify a Q_{rel}.

Wherein: $$Q_{rel} = Q_{axis}\left(10^{\left(\frac{\pm C_{\angle} dB}{10}\right)}\right)$$

Where: $\pm C_{\angle}dB$ indicates the level in dB of the particular angle *relative* to the level in dB on axis.

A complete descriptive may be specified by:

$$Q_{rel} = Q_{axis}\left(10^{\left(\frac{\pm C_{\angle} dB}{10}\right)}\right)(f)$$

Where: f is the frequency at which the measurement is made.

A further useful convention would be to agree that where no "f" is specified then the 1/3 octave band at 2000 Hz is indicated.

In the design of a sound system we use:

$Q_{min(ss)}$

Where: ss stands for single source and which *usually* is synonymous with Q_{axis} but may, on occasion, actually be a Q_{rel}. The term *min* indicates that it is the minimum value that will allow the $\%AL_{cons}$ required at that *point*.

If more than one source is used, we encounter the term:

NQ_{min}

Wherein we increase the Q of the first device proportionately to the number (N) of additional devices (of equal acoustic power output).

We also employ the term Q_{avail} whereby we can calculate the N required for a multiple source system.

$$N = \left(\frac{Q_{min}}{Q_{avail}}\right)$$

A further refinement is the direct calculation of a distance (D_2) at which the Q_{avail} results in the same ratio of direct-to-reverberant sound as NQ_{min} *would* have provided.

$$D_{2\ max} = \left(\frac{D_{2ss}}{N}\right)$$

At the current time we utilize the following Q descriptives:

Q_{axis} Q_{min} Q_{rel} Q_{avail}

along with the descriptive modifiers: ss $\pm C_L dB$ N and f

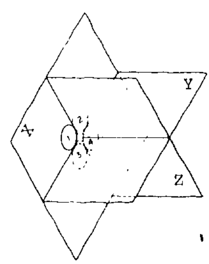
A Subtlety Regarding "Q" by Placement

An often misinterpreted point with regard to establishing a directivity factor (Q) by placement of the source near a reflecting surface (mirror images) is that the source must be at, *not in*, the surface. (See left half of figure.)

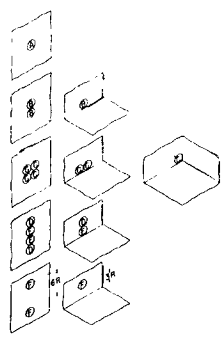
Loudspeakers mounted in the wall will, at lower frequencies, exhibit "mutual coupling" as shown in the right half of the figure.

When a single speaker is mounted *in* a wall, half the power goes into *another* space. When mounted near the wall, half the power is reflected back into the space.

From Henney's HANDBOOK OF ENGINEERING



Primary images 2, 3, and 4 of piston 1 introduced by planes Y and Z.



Effect of adding pistons and reflecting planes on radiation impedance. All pistons marked with the same letter see the same radiation impedance.

ΔD_X and the Use of Q

Users of the Hopkins-Stryker equation often question whether different Qs should be employed for D_S, D_1, D_2 and D_0 when obtaining ΔD_X. Normally, the answer is "use the loudspeaker Q for all distances." Differing Qs (for example, the talker with a Q of 2.5 and the loudspeaker with a Q = 50) will establish quite different D_Cs.

In the normalized gain equations, D_0 drops out and is replaced by EAD. This means that both the ΔD_S and ΔEAD are *normally* in the direct sound field where inverse-square-law level change is the rule and the Q chosen can be arbitrary *so long as it is the same for both distances*. In any case, the remaining two distances, D_1 and D_2, are dependent upon the loudspeaker's Q.

How to Use Differing Values

If it is desired to use separate Q values for the talker and loudspeaker (and perhaps to assign a Q to the microphone as well), you are then freed to obtain an absolute level change rather than a relative one. This is accomplished by using a reference point and a D_X point for each value and taking the ΔD_Xs of both points, followed by using the difference between them as the ΔD_X in the gain equations.

What you may not legitimately do is use differing Qs in the Hopkins-Stryker equation when obtaining *relative* ΔD_Xs. The reference point chosen should usually be less than 0.5 feet.

Using the Hopkins-Stryker Equation

The sound power level (L_W) is referenced to 10^{-12} watt. In using the Hopkins-Stryker equation to obtain an expected sound pressure level (Lp) at some distance (D_X), it is important not to use the L_W of the array but only the L_W of that part of the array supplying L_D and D_X. To do otherwise is to miscalculate the L_D which is dependent *only* upon the L_W of the devices also producing the L_D at the point of observation (measurement). The N factor inserted into the equation in opposition to the total absorption ($S\bar{a}$) correctly adjusts for the ratio of total L_W to the L_W producing L_D because this portion of the Hopkins-Stryker equation affects only the reverberant sound field level (L_R).

Always bear in mind that L_D is affected by that part of L_W producing L_D at the point of measurement (D_X) distance from the array, the Q of the device producing L_D at D_X (not the "Q" of the array), and the distance from the array (D_X). Thus, we avoid the difficulties of overestimating the level of L_D at D_X.

L_R is affected by the *total* L_W of the array (which may be properly accounted for by the ratio N which scales the level appropriately to the L_W of the single device producing L_D) and the total absorption present. Here it is important to note the sometimes significant role of the architectural acoustic modifier (M_a). A substantial M_a can lower L_R (but RT_{60} or the decay rate remains the same). The M_a factor is invariably lower than would be expected because of the difference between the Q of real life devices and the coverage angles employed. It is important to note that this effect cannot operate unless $a_c > \bar{a}$.

Summary

These instructive equations reveal the interaction of each of the primary parameters controlling the various sound fields. We have discussed additional parameters that have direct bearing on the modified behavior of the primary parameters. In its modified form, the Hopkins-Stryker equation has kept pace with measurements in the sense that our prediction accuracy has kept pace with our measurement capability.