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MASTER'S THESIS

Analysis of Structural Damping

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Abstract

One important parameter in the study of dynamic systems is material damping. It is defined as the ability of a material to absorb vibration by internal friction and convert the mechanical energy into heat. In spite of its importance in the study of rotating machinery, it has not been studied deeply because of the difficulty analyzing and testing its different types and mechanisms.

This master thesis presents a new method to measure damping in mechanical systems. The mechanical system analyzed in this essay is a rotating shaft.

The thesis consists of two different parts: a theoretical analysis and experimental work.

Regarding the theoretical analysis, two different methods are explained. The first method is based on the moments, stresses and strains involved in the system and it is the basis to design the experimental set up. The second method is based on the energy and it is proposed to check the validity of the results from the experiment.

On the other hand, two tasks are developed in the experimental work:

- To design the set up for the experiment and assemble it.
- To carry out the experiment with different material in order to check the validity of the proposed method.

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Luleå, December 2005

Manuel Martínez

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Table of symbols

σ	Stress
ε	Strain
$\frac{d}{dt}(\varepsilon_{\max})$	Strain rate
E	Young's Modulus
E*, $\hat{E}$	Complex Modulus
I	Moment of inertia
C_s	
ΔU	Damping capacity
T_p	Peak time
M_p	Peak value
PO	Percentage overshoot
d_v, d_h, d_f	Damping capacity per unit volume
ω	Frecuency
$\dot{q}$	Relative velocity
ρ	Density
C_d	It is a function of the Reynold's number and the geometry
Q	Amplification factor
ζ	Damping ratio
η	Loss factor
γ	Viscosity
T	Time
R	Radius of the shaft

r	Distance between the point and the centre of its section
F_x, F_y,	Forces
M_x, M_y	Moments
dA	Area element
x, y, z	Coordinate
F_b	Applied force
L	Length of the shaft
d	Displacement about x of the shaft
δ	Deformed position
φ	Phase angle

Chapter 1. Introduction

Damping is the energy dissipation of a material or system under cyclic stress. Three main types of damping are present in any mechanical system:

- Internal damping
- Structural damping
- Fluid damping

1.1 Internal damping

Internal damping is caused by microstructure defects -impurities, grain boundaries…-, thermoelastic effects, eddy-current effects in ferromagnetic materials, dislocation motion in metals and chain motion in polymers. Besides, there are two types of internal damping: Viscoelastic damping and hysteretic damping. Dealing with the latter, it must be taken into account that the term, hysteretic damping, is not suitable at all, for the reason that all types of internal damping are related with hysteretic-loop effects. The relation between the stress (σ) and the strain (ε) has a hysteretic loop as the figure (1.1.1) shows.

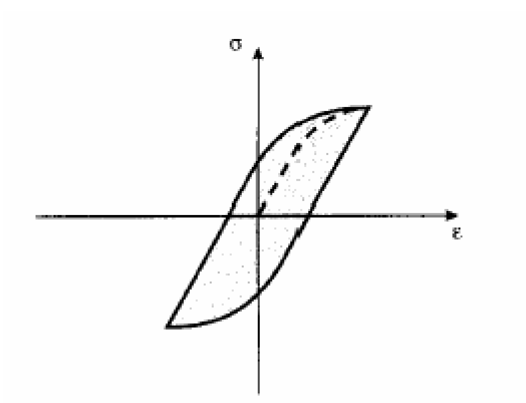

Figure 1.1.1

The area of the hysteretic loop represents the energy dissipated per unit volume of material and per stress cycle. The force and the displacement relationship present also a hysteretic loop.

Concerning **Viscoelastic damping**, the relation between the stress and the strain is expressed through a linear differential equation with respect to time. The stress of a Viscoelastic damping material depends on the frequency of variation of the strain, and therefore on the frequency of motion. What is used to represent Viscoelastic damping is tree models:

- Kelvin-Voigt model: $$\sigma = E \cdot \varepsilon + E^* \cdot \frac{d}{dt}(\varepsilon) \quad (1.1.1)$$
- Maxwel model: $$\sigma + c_s \cdot \frac{d}{dt}(\sigma) = E^* \cdot \frac{d}{dt}(\varepsilon) \quad (1.1.2)$$
- Standard linear solid model: $$\sigma + c_s \cdot \frac{d}{dt}(\sigma) = E \cdot \varepsilon + E^* \cdot \frac{d}{dt}(\varepsilon) \quad (1.1.3)$$

The most widespread model is the Kelvin-Voigt model as it is the most accurate for practical purposes. In equation (1.1.1), E is Young's Modulus and E* is the complex modulus, which is assumed to be time independent. The term Eε represents the elastic behaviour of the material and it does not contribute to damping. The term $E^* \cdot \frac{d}{dt}(\varepsilon)$ is the responsible of damping. It is defined the damping capacity per unit volume as:

$$d_v = E^* \oint \frac{d}{dt}(\varepsilon) \cdot d\varepsilon \quad (1.1.4)$$

If the material is subjected to a harmonic excitation, the strain can be expressed as follows:

$$\varepsilon = \varepsilon_{\max} \cdot \cos(\omega t) \quad (1.1.5)$$

By introducing equation (1.1.5) into equation (1.1.4), it is achieved the following expression for damping capacity per unite volume:

$$d_v = \pi \cdot \omega \cdot E^* \cdot \varepsilon_{\max}^2 \quad (1.1.6)$$

By considering that $\sigma = E \cdot \varepsilon_{\max}$, the following equation is obtained:

$$d_v = \frac{\pi \cdot \omega \cdot E^* \cdot \sigma_{\max}^2}{E^2} \quad (1.1.7)$$

It is possible to say that damping capacity per unit volume for the Kelvin-Voigt model depends on frequency.

The Kelvin-Voigt model will be explained in detail later because it is the chosen model to characterize the Viscoelastic behaviour in this paper.

As regards **hysteretic damping**, the stress does not depend considerably on the frequency of oscillation. What is more, damping capacity per unit volume is independent of frequency as the following expression shows:

$$d_h = J \cdot \sigma_{\max}^n \qquad (1.1.8)$$

For the case n=2, the stress can be expressed by the following expression:

$$\sigma = E \cdot \varepsilon + \frac{\hat{E}}{\omega} \cdot \frac{d}{dt}(\varepsilon) \qquad (1.1.9)$$

This expression is equivalent to equation (1.1.1) by considering that $E^* = \frac{\hat{E}}{\omega}$.

If the material is subjected to a harmonic excitation where the strain is represented also by equation (1.1.1), the equation (1.1.9) can be expressed as follows:

$$\sigma = E \cdot \varepsilon_o \cdot \cos(\omega t) + \hat{E} \cdot \varepsilon_o \cdot \sin(\omega t) = E \cdot \varepsilon_o \cdot \cos(\omega t) + \hat{E} \cdot \varepsilon_o \cdot \cos(\omega t + \frac{\pi}{2}) \qquad (1.1.10)$$

It must be taken into account that the stress consists of the elastic component, which is in phase with strain, and the hysteretic damping component, which is 90º out of phase.

The response can be expressed as follows:

$$\varepsilon = \varepsilon_o \cdot e^{j\omega t} \qquad (1.1.11)$$

By introducing equation (1.1.11) into equation (1.1.10), a new expression for the stress is obtained:

$$\sigma = (E + j\hat{E}) \cdot \varepsilon \qquad (1.1.12)$$

By combining equations (1.1.11) and (1.1.12), an expression, which represents both Viscoelastic and hysteretic damping behaviour, is obtained:

$$\sigma = E \cdot \varepsilon + \left(E^* + \frac{\hat{E}}{\omega} \right) \cdot \frac{d}{dt}(\varepsilon) \qquad (1.1.13)$$

It would be interesting to point out that E, E* and Ê are independent of the frequency.

1.2 Structural damping

Rubbing friction or contact among different elements in a mechanical system causes structural damping. Since the dissipation of energy depends on the particular characteristics of the mechanical system, it is very difficult to define a model that represents perfectly structural damping. The Coulomb-friction model is as a rule used to describe energy dissipation caused by rubbing friction. Regarding structural damping (caused by contact or impacts at joins), energy dissipation is determined by means of the coefficient of restitution of the two components that are in contact.

Structural damping is usually estimated by means of measuring but the measured values represent the total damping in the mechanical system. Consequently it is necessary to estimate the values for the other types of damping and to subtract them from the measured value in order to obtain a value of structural damping. Structural damping is much greater than internal damping and it represents a large portion of energy dissipation in mechanical structures.

As mentioned above, different factors such as rubbing friction or impacts cause structural damping. The most important form of structural damping is the slip damping. This form of damping is caused by Coulomb friction at a structural joint. It depends on many factors such as joint forces or surface properties. Assuming an ideal Coulomb friction, the damping force at a join can be expressed through the following expression:

$$f = c \cdot \mathrm{sgn}\left(\dot{q}\right) \qquad (1.2.1)$$

where:

f = damping force
q = relative displacement at the joint
c= friction parameter
and the signum function is defined by:

sgn(x)= 1 for $x \geq 0$
sgn(x)=-1 for $x < 0$

1.3 Fluid damping

When a material is immersed in a fluid and there is relative motion between the fluid and the material, as a result the latter is subjected to a drag force. This force causes an energy dissipation that is known as fluid damping. The following equation expresses the drag force:

$$f_d = \frac{1}{2} \cdot c_d \cdot \rho \cdot \dot{q}^2 \cdot \operatorname{sgn}\left(\dot{q}\right) \tag{1.3.1}$$

where:

$\dot{q}$ = Relative velocity

ρ = Density

c_d = It is a function of the Reynold's number and the geometry

Damping capacity per unit volume for fluid damping is:

$$d_f = \frac{\oint \int_0^{L_x} \int_0^{L_y} f_d \cdot dz \cdot dx \cdot dq(x, y, z)}{L_x \cdot L_y \cdot q_o} \tag{1.3.2}$$

in which L_x and L_y are cross-sectional dimensions of the element in the x and y direction and q_o is a normalizing parameter for relative displacement.

Chapter 2. Measurement of damping

From a theoretical point of view there are different methods to measure damping. These methods are divided in two main groups depending on if the response of the system is expressed as a function of time or as a function of frequency, i.e. time-response methods and frequency-response methods.

Logarithmic Decrement Method, Step-Response Method and Hysteretic Loop Method are time-response methods, whereas Magnification-Factor Method and Bandwidth Method are frequency-response methods.

Dealing with time-response methods, the **Logarithmic Decrement Method** is the most common method to measure damping. In this method, an initial excitation is applied to a single-degree-of-freedom oscillatory system with viscous damping. As Figure (2.1) shows, the form of the response is a time decay, which is expressed by the Formula (2.1).

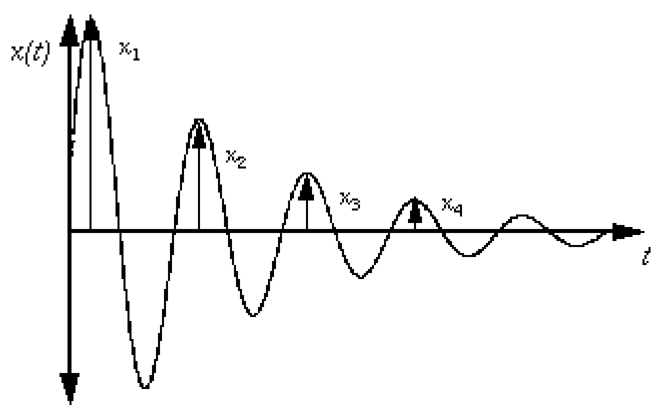

Figure 2.1 Impulse response of a simple oscillator

$$y(t) = y_o \cdot \exp(-\zeta\omega_n t) \cdot \sin(\omega_d t) \qquad (2.1)$$

If the response is known, then it is possible to determine the logarithmic decrement δ through the formula (2.2).

$$\delta = \frac{1}{r} \cdot Ln\left(\frac{X_i}{X_{i+r}}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \qquad (2.2)$$

Then the damping ratio ζ is easily calculated with the formula (2.3).

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \tag{2.3}$$

As regards the **Step-Response Method**, a unit-step excitation is applied to the single-degree-of-freedom oscillatory system and its time response looks like a typical step-response curve (see figure 2.2) given by the expression:

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} \cdot \exp(-\zeta\omega_n t) \cdot \sin(\omega_d t + \varphi) \tag{2.4}$$

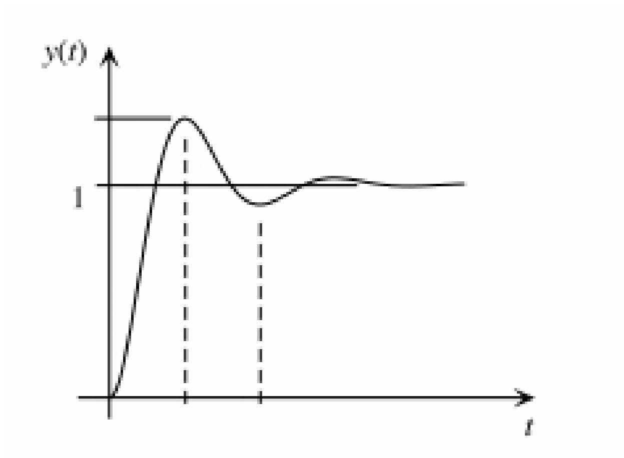

Figure 2.2. A typical step response of a simple oscillator

Damping ratio (ζ) can be determined through these tree parameters: peak time (T_p), peak value (M_p) or percentage overshoot (PO). These tree parameters are easily obtained from the step-response curve. Formulas (2.5), (2.6) and (2.7) show the relation between these tree parameters and damping ratio (ζ).

$$\zeta = \sqrt{1 - \left(\frac{\pi}{T_P \omega_n}\right)} \tag{2.5}$$

$$\zeta = \frac{1}{\sqrt{1 + \frac{1}{\left[\frac{Ln(M_P - 1)}{\pi}\right]^2}}} \tag{2.6}$$

$$\zeta = \frac{1}{\sqrt{1 + \frac{1}{\left[\frac{Ln\left(PO/100\right)}{\pi}\right]^2}}} \tag{2.7}$$

The **Hysteretic Loop Method** calculates the energy loss per cycle of oscillation due to steady state harmonic loading. Damping capacity (ΔU) is given by the area of the displacement-force hysteretic loop. Then, the loss factor (η) and the damping ratio (ζ) can be easily determined through formula (2.8) and (2.9) respectively.

$$\eta = \frac{\Delta U}{2\pi U_{\max}} \tag{2.8}$$

$$\eta = 2\zeta \tag{2.9}$$

The **Magnification-Factor Method** is a frequency-response method, as it has been mentioned above. The damping ratio (ζ) can be determined, on condition that the magnitude curve of the frequency-response function is known (see figure 2.2). From this curve is possible to obtain the amplification factor (Q), which is the magnitude of the frequency-response function at resonant frequency. Then, damping ratio (ζ) can be easily determined using expression (2.10).

$$Q = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \tag{2.10}$$

As a final point, the last method to estimate damping from frequency domain is the **Bandwidth Method.** This method is also based in the magnitude curve of the frequency-response function. Bandwidth (Δω) is defined as the width of the frequency-response magnitude curve when the magnitude is $1/\sqrt{2}$ times the peak value. Then, damping ratio can be determined from bandwidth using the expression (2.11):

$$\zeta = \frac{1}{2}\frac{\Delta\omega}{\omega_r} \qquad (2.11)$$

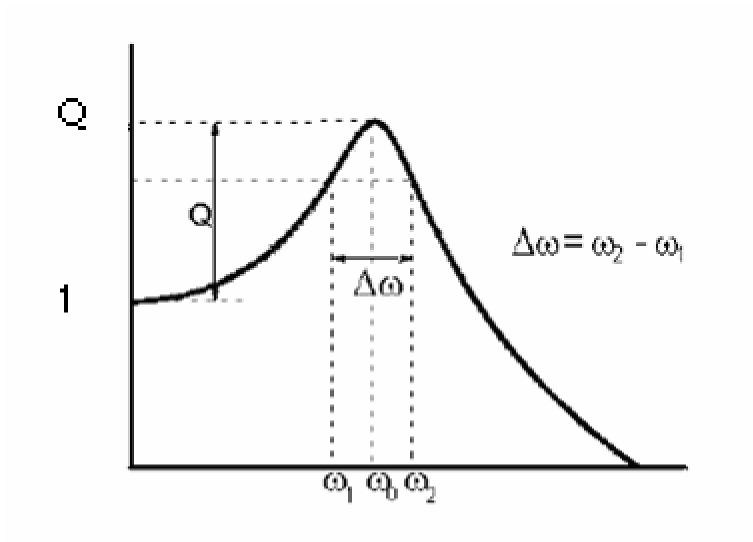

Figure 2.2 Bandwidth method of damping measurement in a single-degree-of-freedom system.

Chapter 3. Measurement of damping in mechanical systems

Although the methods mentioned above are theoretically suitable for measuring damping, it is actually very difficult to apply them for measuring damping in rotating machinery. For this reason, there is a lack of experiments to measure damping in rotating machinery. Consequently, this master thesis tries to design and develop a new experiment to measure damping in mechanical systems.

As it has been pointed out in the introduction, damping can be expressed through different parameters such as damping ratio (ζ) or the loss factor (η). Therefore, the aim of the experiment is to determine one of these parameters by means of measuring simple variables such as forces or displacements. This paper will propose two different methods to measure the loss factor (η).

This thesis consists of two different parts: a theoretical analysis of the problem and an experimental work. The theoretical analysis proposes two different methods to calculate damping. The first method will be used to design the experiment while the second will be used to check the validity of the measured values from the experiment. In the experimental part it will explain how the experiment was carried out.

3.1 Theoretical analysis

To begin with, it is necessary to set up all the conditions involved in the problem. The experiment will focus on a rotating shaft that is simply supported. Moreover, the shaft is subjected to a constant centre load (see figure 3.1.1).

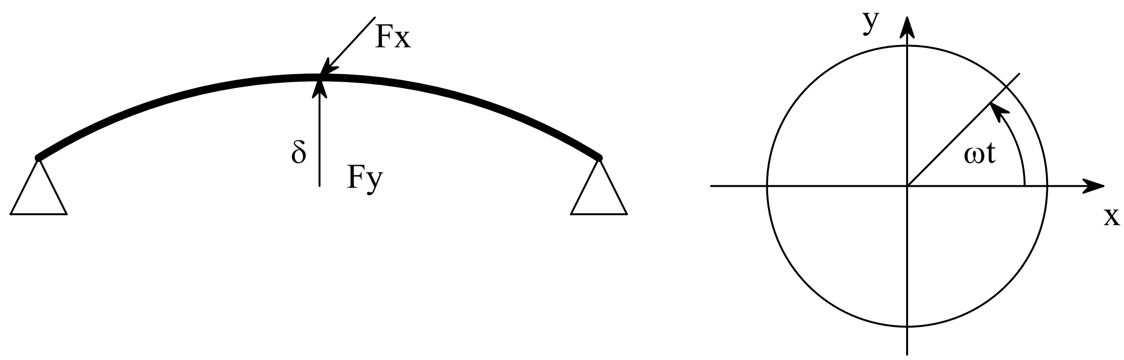

Figure (3.1.1) Moments and forces

One of the most important conditions is the material definition. It must be assumed that the shaft is made of a Viscoelastic material. Viscoelastic materials show simultaneously a viscous and an elastic behaviour. Different models can represent Viscoelastic materials; however in this paper it will only be considered the Kelvin-Voigt Model. This model represents the behaviour of Viscoelastic material by means of a dashpot and a spring as the figure 3.1.2 shows.

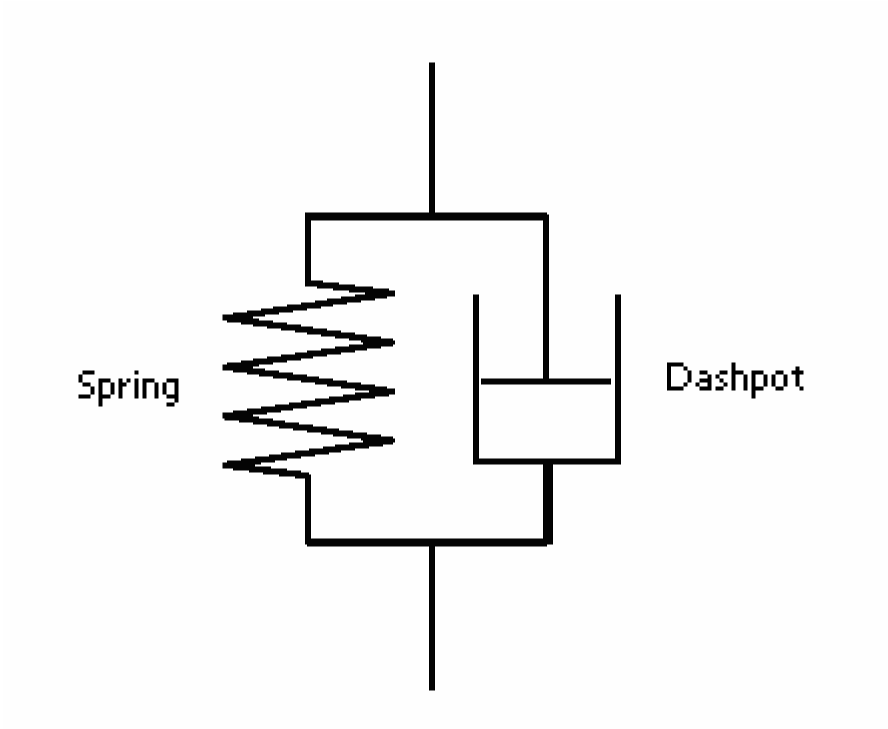

Figure (3.1.2)

The dashpot constrains the spring in order to have the same deformation. The Kelvin-Voigt model is mathematically expressed by the following expression:

$$\sigma = E \cdot \varepsilon + \gamma \cdot \frac{d}{dt}(\varepsilon) \qquad (3.1.1)$$

where:

σ = Stress
ε = Strain
γ = Viscosity
t = time

Two different methods are proposed to measure damping, as it has been mentioned above. The first method tries to relate the loss factor (η) with moments and forces while the second method tries to relate the loss factor (η) with the energy involved in the process.

3.1.1 First Method: Loss factor (η) vs. Moments & Forces

During the experiment, the shaft must be kept in a constant deformed position (δ), as the figure 3.1.1 shows, and rotating with the frequency ω. The strain in any point will change according to the following expression:

$$\varepsilon(r,t) = \frac{\varepsilon_{\max}}{R} \cdot r \cdot \sin(\omega \cdot t) \qquad (3.1.1.1)$$

where:

R = Radius of the shaft
r = Distance between the point and the centre of its section
t = Time
ω = Rotating frequency
$\varepsilon_{\max}$ = Maximum strain

Deriving the equation (3.1.1.1), it is obtained the strain rate, which induces a stationary stress state similar to the bending stress state but rotated 90 degrees. The equation (3.1.1.2) expresses the strain rate:

$$\frac{d}{dt}\varepsilon(r,t) = \frac{\varepsilon_{\max}}{R} \cdot r \cdot \omega \cdot \cos(\omega \cdot t) \qquad (3.1.12)$$

On the other hand, the bending moment about x can be calculated from the following expression:

$$M_x = \int_{-R}^{R} \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{\sigma_{\max} \cdot y}{R} \cdot y \cdot dy \cdot dx \qquad (3.1.1.13)$$

The result of the integration is:

$$M_x = \frac{1}{4} \cdot \sigma_{\max} \cdot R^3 \cdot \pi \qquad (3.1.1.14)$$

M_x can also be expressed through the following expression:

$$M_x = \frac{1}{4} \cdot \sigma_{y\max} \cdot R^3 \cdot \pi \qquad (3.1.1.15)$$

Assuming that:

$$\sigma = \sigma_x + \sigma_y = \gamma \cdot \frac{d}{dt}(\varepsilon) + E \cdot \varepsilon \qquad (3.1.1.16)$$

Then, it would be conclude that M_x is found as follows:

$$M_x = \frac{1}{4} \cdot E \cdot \varepsilon_{\max} \cdot R^3 \cdot \pi \qquad (3.1.1.17)$$

In the same way than with M_x, it is possible to reason in order to obtain M_y:

$$M_y = \int_{-R}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \frac{\sigma_{\max} \cdot x}{R} \cdot x \cdot dx \cdot dy \qquad (3.1.1.18)$$

The result of the integration is:

$$M_y = \frac{1}{4} \cdot \sigma_{\max} \cdot R^3 \cdot \pi \qquad (3.1.1.19)$$

Equation (3.1.1.19) can be expressed just as equation (3.1.1.20) shows:

$$M_y = \frac{1}{4} \cdot \sigma_{x\max} \cdot R^3 \cdot \pi \qquad (3.1.1.20)$$

Reasoning in a similar way than with the moment about x, it is obtained an expression that depends on the viscosity, the strain rate and the radius (3.1.1.20).

$$M_y = \frac{1}{4} \cdot \gamma \cdot \frac{d}{dt}(\varepsilon_{\max}) \cdot R^3 \cdot \pi \qquad (3.1.1.21)$$

At this point, an expression that relates a moment with the viscosity has been obtained. However, the aim is to get an expression that relates a moment or a force with the loss factor (η). Thus, the next step will be to relate the viscosity with the loss factor (η). It will be used the complex modulus to relate both parameters.

As it has been mentioned above, Viscoelastic materials show simultaneously a viscous and an elastic behaviour. It means that their modulus must be represented by a complex quantity, which possesses both stored and dissipative energy component. This complex modulus can be expressed in different ways depending on the model used to describe the system. On one hand, one of the hysteretic models defines the complex modulus through the following expression:

$$E^* = E(1 + i\eta) = E + i \cdot \eta \cdot E \qquad (3.1.1.22)$$

On the other hand, the hysteretic model can also be applied when a material is subjected to cyclic loading. In this case, it is necessary to assume that both the time history of the stress cycles and the history of the deformation are harmonic. Moreover, the strain will be delayed in time by a phase angle φ, which is considered independent from the frequency. Under these conditions the time histories of stress, strain and strain rate can be expressed as follows:

$$\sigma = \sigma_o \cdot \cos(\omega \cdot t) = \sigma_o \cdot e^{i\omega \cdot t} \qquad (3.1.1.23)$$

$$\varepsilon = \varepsilon_o \cdot \cos(\omega \cdot t - \varphi) = \varepsilon_o \cdot e^{-i \cdot \varphi} \cdot e^{i \cdot \omega \cdot t} \qquad (3.1.1.24)$$

$$\frac{d}{dt}(\varepsilon) = \varepsilon_o \cdot e^{-i \cdot \varphi} \cdot e^{i \cdot \omega \cdot t} \cdot i \cdot \omega \qquad (3.1.1.25)$$

where:

ω: frequency of the cyclic loading
t: time
φ: phase angle
σ_o: maximum stress
ε_o: maximum strain

The complex modulus relates stress and strain according to the following expression:

$$E^* = \frac{\sigma}{\varepsilon} \qquad (3.1.1.26)$$

By introducing equations (3.1.1.23) and (3.1.1.24) into equation (3.1.1.26), the following expression for the complex modulus is obtained:

$$E^* = \frac{\sigma_o}{\varepsilon_o \cdot e^{-i\cdot\varphi}} \tag{3.1.1.27}$$

Now, it is necessary to relate σ_o with ε_o. By introducing the equations (3.1.1.24) and (3.1.1.25) into equation (3.1.1), the following expression is obtained:

$$\sigma = E \cdot \left(\varepsilon_o \cdot e^{-i\cdot\varphi} \cdot e^{i\cdot\omega\cdot t}\right) + \gamma \cdot e^{-i\cdot\varphi} \cdot e^{i\cdot\omega\cdot t} \cdot i \cdot \omega \tag{3.1.1.28}$$

As equations (3.1.1.23) and (3.1.1.28) define the same parameter –stress- it is possible to state that:

$$E \cdot \left(\varepsilon_o \cdot e^{-i\cdot\varphi} \cdot e^{i\cdot\omega\cdot t}\right) + \gamma \cdot e^{-i\cdot\varphi} \cdot e^{i\cdot\omega\cdot t} \cdot i \cdot \omega = \sigma_o \cdot e^{i\omega\cdot t} \tag{3.1.1.29}$$

Consequently, σ_o can be expressed through the following expression:

$$\sigma_o = \varepsilon_o \cdot e^{-i\cdot\varphi} \cdot (E + i \cdot \omega \cdot \gamma) \tag{3.1.1.30}$$

If equation (3.1.1.30) is introduced into equation (3.1.1.27), it is obtained an expression, which defines the complex modulus:

$$E^* = E + i \cdot \gamma \cdot \omega \tag{3.1.1.31}$$

If equations (3.1.1.22) and (3.1.1.31) are compared, it is possible to assert that:

$$\gamma \cdot \omega = E \cdot \eta \tag{3.1.1.32}$$

Therefore, viscosity can be expressed as follows:

$$\gamma = \frac{E \cdot \eta}{\omega} \tag{3.1.1.33}$$

The equation (3.1.1.33) is especially interesting because it shows that viscosity depends on the frequency.

After relating the viscosity with the loss factor (η), the last step would be to relate the loss factor (η) with a force instead of a moment because measuring moments can be more complicated that measuring forces.

At this point, it would be possible to express M_y as function of the loss factor (η) by introducing equation (3.1.1.33) into equation (3.1.1.21).

$$M_y = \frac{1}{4} \cdot \frac{E \cdot \eta}{\omega} \cdot \frac{d}{dt}(\varepsilon_{max}) \cdot R^3 \cdot \pi \qquad (3.1.1.34)$$

Applying the solid mechanics theory to the shaft shown in figure (3.1.1), it is possible to state that:

$$F_x = \frac{4}{L} \cdot M_y \qquad (3.1.1.35)$$

$$F_y = \frac{4}{L} \cdot M_x \qquad (3.1.1.36)$$

where L is the length of the shaft.

One of the parameters of equation (3.1.1.34), $\varepsilon_{máx}$, is unknown; hence it is necessary to determine it. It is possible to express $\varepsilon_{máx}$ as function of F_y. The value of F_y is known because it is the force that is introduced to keep the shaft in a constant deformed position (δ). Introducing equation (3.1.1.17) into equation (3.1.1.26), it can be obtained the next expression:

$$\frac{1}{4} \cdot E \cdot \varepsilon_{max} \cdot R^3 \cdot \pi = \frac{L}{4} \cdot F_y \qquad (3.1.1.37)$$

By working out the value of $\varepsilon_{máx}$ from equation (3.1.1.37), the following expression is obtained:

$$\varepsilon_{max} = \frac{L \cdot F_y}{E \cdot R^3 \cdot \pi} \qquad (3.1.1.38)$$

Finally, the last thing would be to relate the loss factor (η) with the parameters that are known or that can be easily measurable. By introducing the equation (3.1.1.34) into equation (3.1.1.35), the following expression is obtained:

$$F_x = \frac{4}{L}\frac{1}{4}\cdot\frac{E\cdot\eta}{\omega}\cdot\frac{d}{dt}(\varepsilon_{\max})\cdot R^3\cdot\pi \qquad (3.1.1.39)$$

If equation (3.1.1.38) is introduced into equation (3.1.12), $\frac{d}{dt}(\varepsilon_{\max})$ is expressed through the following:

$$\frac{d}{dt}(\varepsilon_{\max}) = \frac{\frac{L\cdot F_y}{E\cdot R^3\cdot\pi}}{R}\cdot r\cdot\omega\cdot\cos(\omega\cdot t) \qquad (3.1.1.40)$$

Thus, F_x can be expressed through the following expression:

$$F_x = \frac{4}{L}\frac{1}{4}\cdot\frac{E\cdot\eta}{\omega}\cdot\frac{\frac{L\cdot F_y}{E\cdot R^3\cdot\pi}}{R} r\cdot\omega\cdot\cos(\omega\cdot t)\cdot R^3\cdot\pi \qquad (3.1.1.41)$$

By simplifying equation (3.1.1.41), the following expression is obtained:

$$F_x = \frac{r}{R}\cdot F_y\cdot\eta\cdot\cos(\omega\cdot t) \qquad (3.1.1.42)$$

As F_x and F_y will be measured in the outer part of the shaft, it is possible to state that:

$$F_x = F_y\cdot\eta\cdot\cos(\omega\cdot t) \qquad (3.1.1.43)$$

As the value of F_x is fluctuating in time, it is possible to assert that the maximum value of F_x is given by the following expression:

$$F_x = F_y\cdot\eta \qquad (3.1.1.44)$$

As a result, the loss factor (η) can be calculated as function of F_y and the maximum value of F_x:

$$\boxed{\eta = \frac{F_x}{F_y}} \qquad (3.1.1.45)$$

3.1.2 Second Method: Damping energy

Assuming that a harmonic load, $F_b \sin(\omega t)$, at the centre of a simply supported beam of length L, the damping energy can be integrated according to:

Elastic energy: $$W_e = \iint F_b \cdot \varepsilon \cdot dz \cdot dA \qquad (3.1.2.1)$$

Damping energy: $$W_d = \iiint F_b \cdot \left(\frac{d}{dt} \varepsilon \right) \cdot dz \cdot dA \cdot dt \qquad (3.1.2.2)$$

Where:

dA: area element
z: coordinate along the shaft
F_b: applied force
ε: strain

Using the formulas above the damping energy over a period becomes:

$$W_d = \int_0^{\frac{2\cdot\pi}{\omega}} \int_{-R}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \int_0^L \gamma \cdot \left(\frac{y \cdot z \cdot F_b}{E \cdot I} \cdot \omega \cdot \cos(\omega \cdot t) \right)^2 \cdot dx \cdot dy \cdot dz \cdot dt \qquad (3.1.2.3)$$

The solution of the integration is given by the following expression:

$$W_d = \frac{1}{12 \cdot E \cdot I^2} \cdot L^3 \cdot \pi^2 \cdot \omega \cdot \gamma \cdot F_b^{\ 2} \cdot R^4 \qquad (3.1.2.4)$$

The maximum elastic energy is expressed through the following expression:

$$W_e = \int_{-R}^{R} \int_{-\sqrt{R^2-y^2}}^{\sqrt{R^2-y^2}} \int_0^L \left(\frac{y \cdot z \cdot F_b}{E \cdot I} \right)^2 \cdot dz \cdot dx \cdot dy \qquad (3.1.2.5)$$

Having solved the integration above, an expression for the elastic energy is obtained:

$$W_d = \frac{1}{12 \cdot E \cdot I^2} \cdot L^3 \cdot F_b^{\ 2} \cdot R^4 \cdot \pi \qquad (3.1.2.6)$$

The loss factor (η) is defined as the ratio of damping energy loss per radian divided with the peak strain energy (elastic) as follows:

$$\eta = \frac{W_d}{2 \cdot \pi \cdot W_e} \tag{3.1.2.7}$$

By introducing equations (3.1.2.5) and (3.1.2.6) into equation (3.1.2.7), an expression for the loss factor (η) is obtained:

$$\boxed{\eta = \frac{\omega \cdot \gamma}{2 \cdot E}} \tag{3.1.2.8}$$

3.2. Experimental work

This master thesis was initially conceived as a research where the experimental work should have been very important. However, the poor results during experiments changed the aim of this master thesis. Since it was not possible to carry out successful experiments to confirm the validity of the proposed method, it was decided that this thesis should analyse why the experiment was a failure.

First of all, it will be described the setup of the experiment. It consists basically of a simply supported beam that is connected through a coupler to an electric motor. The beam is supported by two ball bearings that provides only translational constrains. This means that reaction forces may be induced but not moments. This group of elements is mounted over a metallic frame as shows figure 3.2.1.

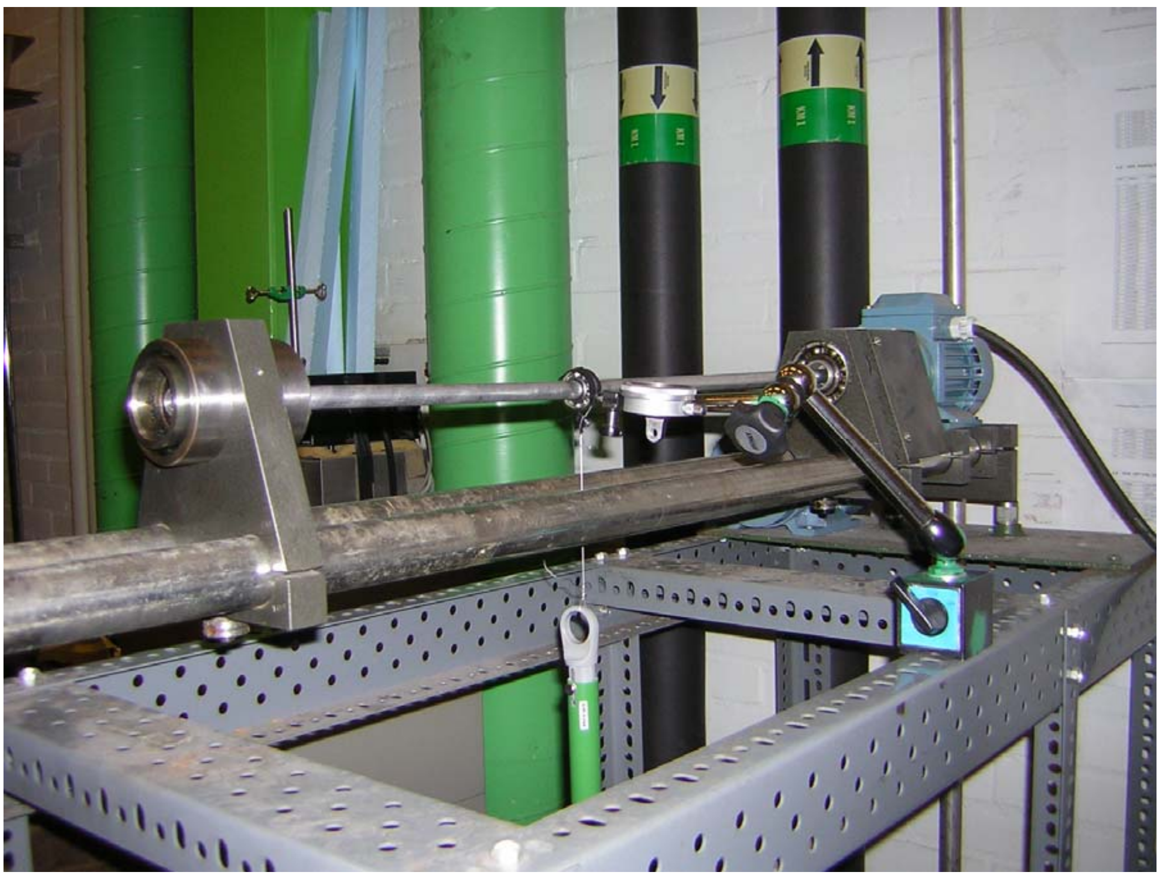

Figure 3.2.1

As it has been mentioned in the theoretical analysis part, the loss factor (η) can be calculated as function of F_y and the maximum value of F_x. It is important to remember that F_y is the introduced force to keep the shaft in a constant deformed position (δ). The easiest way to get the loss factor (η) would be to measure F_x and F_y directly. However, faced with the shortage of resources, it was decided to obtain F_y through a direct measurement and F_x through an indirect measurement.

As regards F_y, it is necessary to apply a force to the shaft and, at the same time, measure it. To achieve this purpose was used a dynamometer. The dynamometer was attached to the frame and the beam as figure 3.2.2 shows.

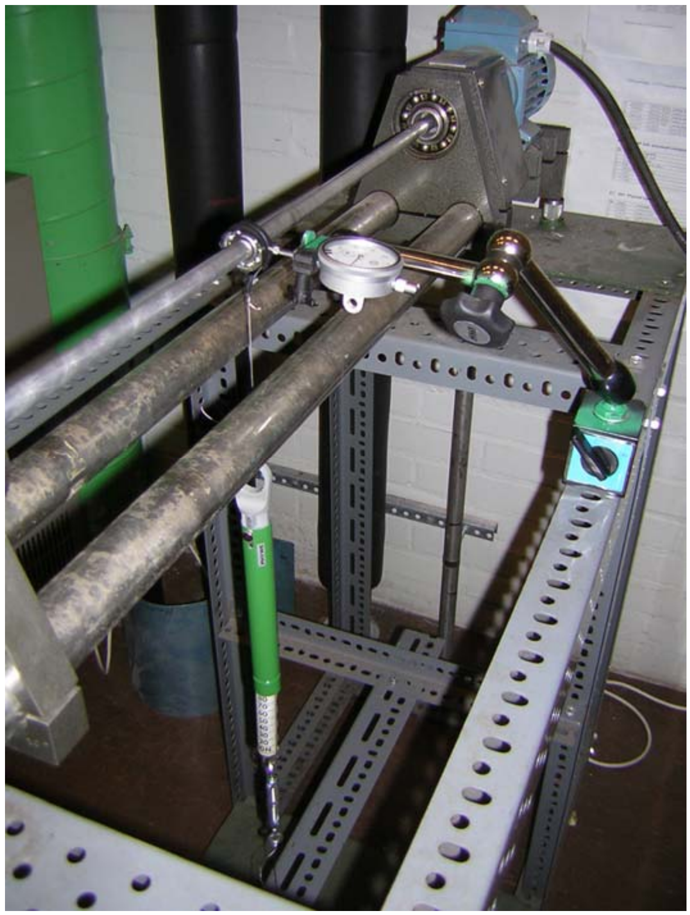

Figure 3.2.2

Nevertheless, it was especially difficult to attach the dynamometer to the beam because the beam should be rotating during the experiment. Using a ball bearing, as figure 3.2.3 shows, the problem is solved. The bearing was attached to the dynamometer through a metallic wire.

Figure 3.2.3

The following picture shows the dynamometer and the device that was used to adjust the introduced force.

Figure 3.2.4

About F_x, it was considered that it could be obtained by relating it with F_y through geometry. Assuming that L is the constant distance between the beam and the place where the dynamometer is attached to frame and *d* is the displacement of the beam about x, then if is possible to obtain the value of F_x by applying geometry (see figure 3.2.5)

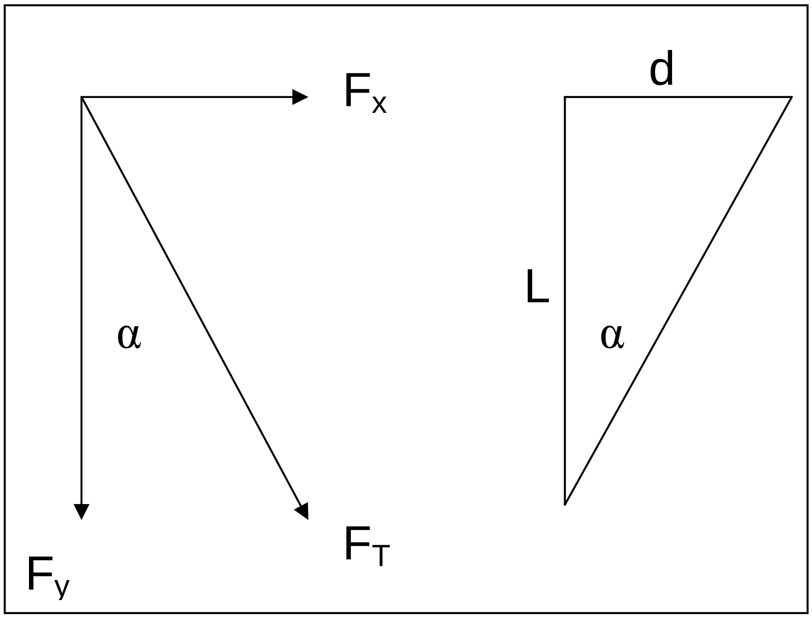

Figure 3.2.5

From the figure xxx it is possible to infer the followings expression:

$$tg\alpha = \frac{d}{L} = \frac{F_x}{F_y} \tag{3.2.1}$$

As it was demonstrated previously in the theoretical part, the loss factor (η) can be expressed through the following equation:

$$\eta = \frac{F_x}{F_y} \tag{3.2.2}$$

By identifying terms in equations (3.2.1) and (3.2.2), it is possible to assert that:

$$\eta = \frac{d}{L} \tag{3.2.3}$$

Surprisingly, the loss factor (η) can be easily obtained by measuring only a length (L) and a displacement (d). As it was said previously, the length is given by the geometry of the experimental set up. So, it is only necessary to measure the displacement about x of the beam (d).

Analyzing the best way to measure this displacement and the available equipment, it was decided that the best option was to use a dial indicator that is shown in figure 3.2.6.

Figure 3.2.6

The range of the dial indicator is from 0 to 10 mm and its graduation 0.01 mm. To hold the dial indicator it was used a magnetic base indicator holder.

Chapter 4. Discussion

This part should show and discuss data from the experiments. However, the experiments failed because of the lack of equipment. Therefore, next it will be analyzed the different reasons that make impossible to obtain good results.

Firstly, it must be taken into account that the forces involved in the problem as well as stresses or strains are expressed through trigonometric functions. It means that they have a periodic and fluctuating behaviour and thus, the displacement (d) will behave in the same way. Since the dial indicator was analogical, it was very difficult to determine with precision the range of fluctuating displacements because their values changed too fast.

Secondly, it was observed that the vibration on the beam was greater than the initially expected. It could be due to the fact that the beam had initially a permanent bend shape, what caused a higher level of vibration and thus larger displacements of the beam. It means that it is complicate to evaluate the amount of the displacement (d) that is the main reason for this additional vibration and to the normal vibration.

Thirdly, it was assumed that the length L was constant but actually it was not. As it was pointed out above, the deformed position (δ) of the beam was obtained by attaching the beam to the frame through a dynamometer, a metallic wire and a tightener. It is well known that the dynamometer is a device that is based on the deformation of a spring and that this deformation is proportional to the applied force (Newton's Law). During the experiment it was observed that there were small displacements, both vertical and horizontal, in the group composed of the dynamometer, the wire and the tightener. It was due to the vibration transmitted from the rotating shaft to the frame.

Chapter 5. Conclusions

This master thesis has established a new theoretical model to measure damping in mechanical systems.

The theoretical analysis indicates that it is possible to measure damping according to the proposed experiment.

The experiment failed due to the lack of suitable equipment. Nevertheless, it should be possible to carry out successful experiments using better facilities.

Finally, it is considered that this master thesis could be a good point of departure for further works.

Chapter 6. Recommendations for further work

Since the proposed method could not be proved, the most part of the recommendations are related with the experimental work.

Firstly, the dial indicator should be digital because it should be very useful to connect it to a computer in order to analysis how the displacement (d) depends on the frequency or when the displacement (d) is maximum.

Secondly, the displacement (d) could be measured by a laser displacement sensor or by a non-contact inductive displacement sensor in order to get more accurate results.

Thirdly, to reduce vibration problems, the shaft should be attached to the frame through a rigid element. In this way, it could be assumed that the length (L) is really constant.

To conclude, the theoretical analysis of this thesis is based on a single degree of freedom system model. Using a multi-degree of freedom system, the results should give better and more realistic results.

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