Engineering News

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1515 S. Manchester Avenue, Anaheim, California

TECHNICAL LETTER NO. 207

A MORE ACCURATE WAY OF CALCULATING D_c

by

Don Davis

In finding the distance at which the direct sound energy is equal to the reverberant sound energy, we have been using the formulas:

$$D_c = 0.14 \sqrt{QS\bar{a}}$$

Where: D_c = critical distance in ft

Q = sound source directivity factor

S = total boundary surface area in ft^2

$\bar{a}$ = the average absorption coefficient

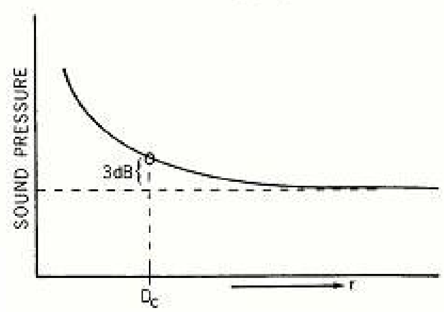
$$\bar{a} = \frac{s_1 a_1 + s_2 a_2 + \ldots . s_n a_n}{S}$$

Where: s = individual surface area

a = absorption of individual surface area

S = total boundary surface area

In actual cases sound does not exactly follow the inverse square law attenuation rate until D_c and then stop. The real life case looks more like the curve shown below.



RELATIVE ATTENUATION FOR ANY GIVEN r

The formula used to calculate the relative attenuation at any given r is:

$$-10 \text{ Log}_{10}\left[\left(\frac{Q}{4 \pi r^2}\right) + \left(\frac{4}{R}\right)\right]$$

Where: r = distance from source in ft

Q = directivity factor of sound source

$R = \frac{S\bar{a}}{1-\bar{a}}$

Mr. Melvin Sprinkle and Mr. John Jenkins have independently derived a more accurate method of obtaining D_c, especially when $\bar{a}$ is a higher value as is often the case in less reverberant, smaller rooms.

Their formula is:

$$D_c = 0.141 \sqrt{QR}$$

For example, using both the old formula and the new formula on a room with 16,600 ft^2 of boundary surface area, D_c would vary with $\bar{a}$ as follows:

	Using $0.14\sqrt{QS\bar{a}} = D_c$	Using $0.141\sqrt{QR} = D_c$
Where:	$\bar{a} = 0.1$, $D_c = 12.75'$	$\bar{a} = 0.1$, $D_c = 13.54'$
	$\bar{a} = 0.2$, $D_c = 18.04'$	$\bar{a} = 0.2$, $D_c = 20.31'$
	$\bar{a} = 0.3$, $D_c = 22.09'$	$\bar{a} = 0.3$, $D_c = 26.59'$
	$\bar{a} = 0.4$, $D_c = 25.51'$	$\bar{a} = 0.4$, $D_c = 33.17'$
	$\bar{a} = 0.5$, $D_c = 28.52'$	$\bar{a} = 0.5$, $D_c = 40.62'$
	$\bar{a} = 0.6$, $D_c = 31.24'$	$\bar{a} = 0.6$, $D_c = 49.75'$

Assume Q = 5

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For those of you interested in the full derivation of this more accurate critical distance formula, it is given below. Our thanks are extended to both Mr. Sprinkle and Mr. Jenkins for their thorough scholarship and thoughtfulness in communicating their results to us.

DERIVATION OF CRITICAL DISTANCE FORMULA

The critical distance is defined as the distance from a sound source in a room with some reverberation, at which the direct sound field and the reverberant sound fields are equal. It is obtained from the Hopkins-Stryker-Beranek formula for relative sound pressure level:

(Eq 1)

$$\text{Relative SPL} = 10 \text{ Log } 10\ (Q/4\pi r^2 + 4/R)$$

For a given Room Constant, R, the limiting (minimum) value of the relative sound pressure level asymptotically approaches a value $10 \text{ Log}_{10}(4/R)$. This limiting value is that of the reverberant sound field in the room of Room Constant, R.

Since the direct sound field follows an inverse square law loss, its loss at the critical distance is:

(Eq 2)

$$\text{Relative SPL} = 20 \log_{10}\left(\frac{1}{D_c}\right) + 10 \text{ Log}\left[\left(\frac{Q}{4\pi r^2}\right) + \left(\frac{4}{R}\right)\right]$$

It will be noted that equation 2 has two terms. The first is the pure inverse square loss between D_c and one foot; the second term, for $r = 1$ gives the absolute loss at one foot.

Since by definition the reverberant and direct fields are equal at D_c, it follows that:

(Eq 3)

$$10 \text{ Log}_{10}\left(\frac{4}{R}\right) = 20 \text{ Log}_{10}\left(\frac{1}{D_c}\right) + \text{Log}_{10}\left(\frac{Q}{4\pi} + \frac{4}{R}\right)$$

(Eq 4)

$$10 \text{ Log}_{10}\left(\frac{4}{R}\right) = 10 \text{ Log}_{10}\left(\frac{1}{D_c^2}\right) + 10 \text{ Log}_{10}\left(\frac{Q}{4\pi} + \frac{4}{R}\right)$$

Now that we have an equation with 10 Log on both sides, we can discard the logs, remembering that the two terms on the right must be multiplied:

(Eq 5)

$$\frac{4}{R} = \left(\frac{1}{D_c^2}\right)\left(\frac{Q}{4\pi} + \frac{4}{R}\right)$$

We can now find a common denominator for the second term and combine:

(Eq 6)

$$\frac{4}{R} = \left(\frac{1}{D_c^2}\right)\left(\frac{RQ + 16\pi}{4\pi R}\right) = \frac{RQ + 16\pi}{4\pi R\ D_c^2}$$

We can now cross multiply, factor and simplify:

(Eq 7)

$$16\pi \not{R}\, D_c^2 = R^2Q + 16\pi R = \not{R}\,(RQ + 16\pi)$$

(Eq 8)

$$D_c^2 = \frac{RQ + 16\pi}{16\pi} = \frac{RQ}{16\pi} + 1$$

Since RQ is almost always much larger than 1, we can discard the 1 term with little error and solve for D_c:

(Eq 9)

$$D_c = \sqrt{\frac{RQ}{16\pi} + 1} = \sqrt{\frac{RQ}{16\pi}}$$

Using the H-P 9100 Computing Calculator, we can bring the 16π outside the radical as a constant:

$$16\pi = 50.26548 \qquad 1/16\pi = 0.01989$$

$$\sqrt{1/16\pi} = 0.14105$$

Therefore:

(Eq 10)

$$\boxed{D_c = 0.141\sqrt{RQ}}$$

Where: D_c is critical distance

R is the room constant

Q is the loudspeaker directivity factor