



TECHNICAL LETTER NO. 211

NOTES ON LOUSPEAKER DIRECTIVITY

To date, ALTEC is the only manufacturer in the world to specify the directivity of its loudspeakers. Two earlier technical letters (Technical Letter 183 — Comments on Directivity of Loudspeakers and Microphones) and Technical Letter 201A — Useful Directivity Data for Reverberant Sound Fields) pioneered the use of such data by sound contractors.

The need of Q is so universal as to be almost taken for granted. It is used in the calculation of the following factors:

1. The sensitivity of a given driver-horn combination in the free field.
2. The critical distance.
3. The ratio of direct to reverberant sound.
4. The efficiency of the loudspeaker.
5. The maximum distance from the loudspeaker for an acceptable articulation loss for consonants.
6. The distribution of the loudspeaker's energy over a given area.
7. The directivity index of the loudspeaker.
8. The attenuation with increasing distance from the loudspeaker in enclosed spaces.

What is Q?

The dimensionless number Q is the ratio of a given area on a sphere (representing the solid radiation angle of a horn) to the total area of the sphere. EXAMPLE: Consider a sphere with a radius of four feet for this discussion, four feet is chosen because that is also the distance at which loudspeaker sensitivity is measured on ALTEC loudspeakers. Any radius could be used in discussing Q by itself). The total surface area is found by $4 \pi r^2$ which is 201.0619298 ft^2 .

Because this area divided by itself equals 1, the Q for a perfect spherical radiator = 1 (see Figure 1).

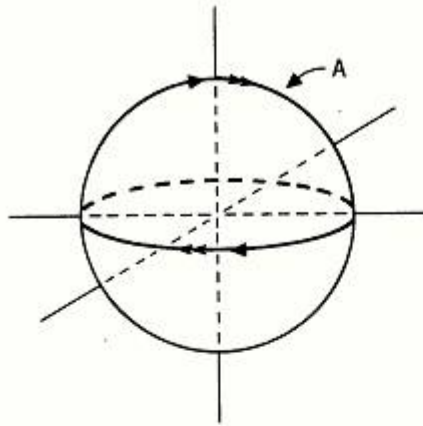


Figure 1. Q = 1

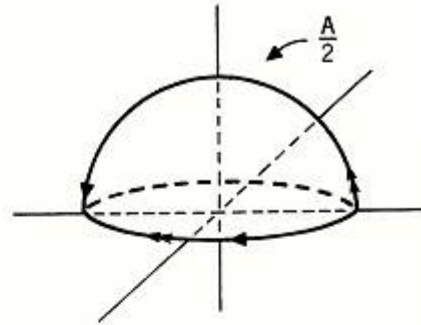


Figure 2. Q = 2

If this sphere is cut in half, a vertical angle of 180° by a horizontal angle of 180° can be evolved from a point at the center of the sphere. The hemispherical surface area is therefore:

$$\frac{201.0}{2} = 100.5 \text{ ft}^2$$

and

$$\frac{201.0}{100.5} = 2 = Q$$

Q = 2 describes a hemispherical radiator (see Figure 2).

If the hemisphere is cut in half, a horizontal angle of 180° and a vertical angle of 90° can then be measured from the center point of the sphere. This provides $1/4$ the surface area of the original sphere, (see Figure 3) or an area of 50.2654825 ft^2 .

$$\frac{201.0619298}{50.2654825} = 4$$

Therefore:

$$Q = 4$$

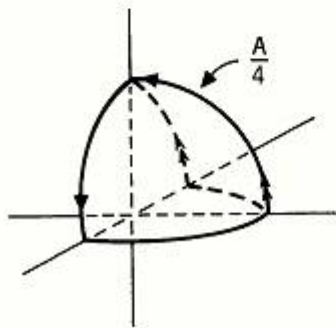


Figure 3. Q = 4

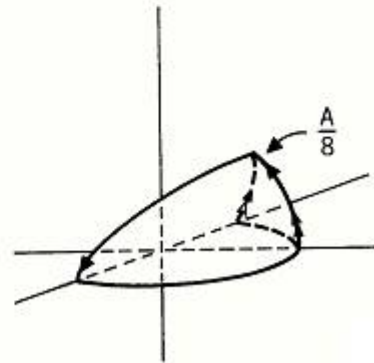


Figure 4. Q = 8

If the section is halved again so that angles of 180° by 45° are obtained (see Figure 4), the area becomes 25.1327412 ft^2 .

$$\frac{201.0619298}{25.1327412} = 8$$

Therefore: $Q = 8$

Calculation of a Horn's Theoretical Q

The shape shown in Figure 5 closely approximates that of many horns. From experience with Technical Letter 201-A, it was found Q varies with frequency. For this discussion, assume Q does not vary with frequency. The following formula WILL accurately compute Q for the angles shown:

$$Q = \frac{180}{\sin^{-1} \left(\sin \frac{\alpha}{2} \cdot \sin \frac{\beta}{2} \right)} \quad *$$

*C. T. Mullan - Calculation of the Directivity Index for Various Types of Radiators J. Acoust. Soc. Amer., 20:387-405 (1948).

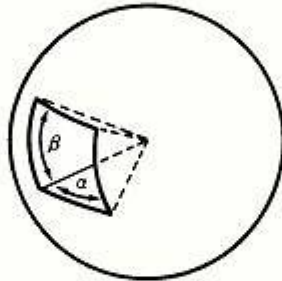


Figure 5. Radiation into a Solid Cone of Space Defined by the Angles α and β

Once the Q number is determined, the directivity index (DI) can be found quickly:

$$DI = 10 \log_{10} Q$$

Figure 6 shows how Q and DI vary with differing combinations of angles between 180° and 40° .

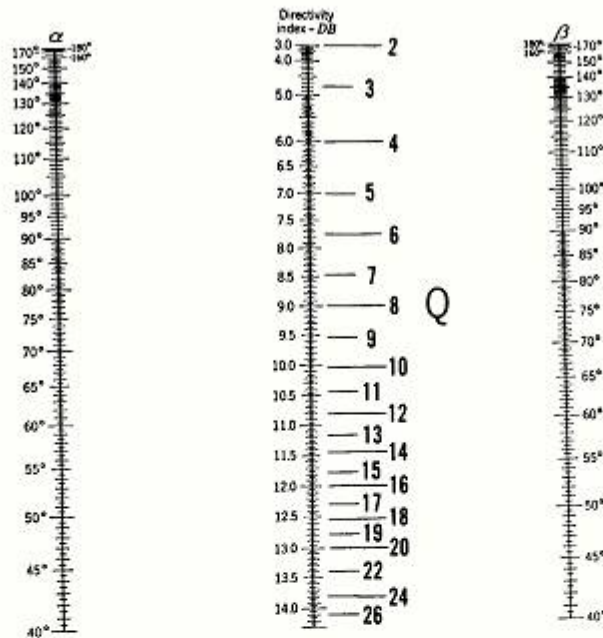


Figure 6. Variation of Q and DI with Different Angle Combinations

Directivity Index

Imagine a loudspeaker capable of perfect spherical radiation and being 100% efficient i.e. for one electrical watt input, one acoustical watt came out. If this loudspeaker were placed at the center of the sphere (radius 4') and a measuring microphone were placed anywhere within four feet of it at any angle, the following SPL_S would be measured:

$$SPL_S = (10 \log_{10} \frac{W10^{13}}{4 \pi r^2}) + 0.5 \text{ dB}$$

SPL_S = the sound pressure level of a spherical radiator

W = electrical watts input to the radiator

r = the radius of the sphere

Therefore:

$$SPL_S = (10 \log_{10} \frac{1 \times 10^{13}}{4 \pi 16}) + 0.5 = 107.47 \text{ dB SPL}$$

This means that at 4' with one electrical watt of input power, a 100% efficient loudspeaker would measure 107.47 dB SPL.

Assuming previous conditions with the single exception that instead of a Q = 1, it now has a Q = 6 (90° x 90°); the formula then becomes:

$$SPL_{\theta} = (10 \log_{10} \frac{WQ}{4 \pi r^2} \cdot 10^{13}) + 0.5 \text{ dB}$$

SPL_{θ} = the sound pressure level of an angular radiator

W = electrical watts input to the radiator

r = the distance from the source to the measurement

Therefore:

$$SPL_{\theta} = (10 \log_{10} \frac{1 \times 6 \times 10^{13}}{4 \pi 16}) + 0.5 = 115.25 \text{ dB SPL}$$

This means that with all conditions identical, the narrowing of the beam of sound increases the SPL available over the area covered by the beam.

90° x 90° beam of sound	= 115.25 dB
Spherical beam of sound	= 107.47 dB
Difference	= <u>7.78 dB</u>

Figure 6 shows that this is the DI for 90° x 90°. If this sound beam were narrowed to 46.5° x 46.5° and using the DI scale in Figure 6 the DI is found to be 13.0 dB:

46.5° x 46.5°	= 13.00 dB
90° x 90°	= 7.78 dB
Difference	= <u>5.22 dB</u>

The increased sensitivity rating for a given driver as it is attached to narrower and narrower horns can be predicted by referring to DI.

Real Life Use of Q

All of the above is theoretical. Real horns do not have precisely defined patterns. Sometimes they have side lobes and back lobes. Changing to narrower horns may actually change the basic efficiency because of difference in throat designs, etc.

ALTEC contractors have real life data to design from. Technical Letter 203 documents real measured sensitivity figures. Technical Letter 201-A documents real measured Q figures. These data can be converted easily into accurate DI figures, efficiency figures, etc.

Calculating the Q of a Loudspeaker in a Room

Preferably pick a diffuse, reverberant space with easily calculated total internal volume and total boundary surface area. Measure the RT_{60} for a band of noise from 500 Hz to 2000 Hz and use the 9067B filter set as shown in Figure 7.

Use $\bar{\alpha}$ to calculate R:

$$\bar{\alpha} = 1 - e^{-\left(\frac{0.049V}{S \cdot RT_{60}}\right)}$$

$$R = \frac{S\bar{\alpha}}{1-\bar{\alpha}}$$

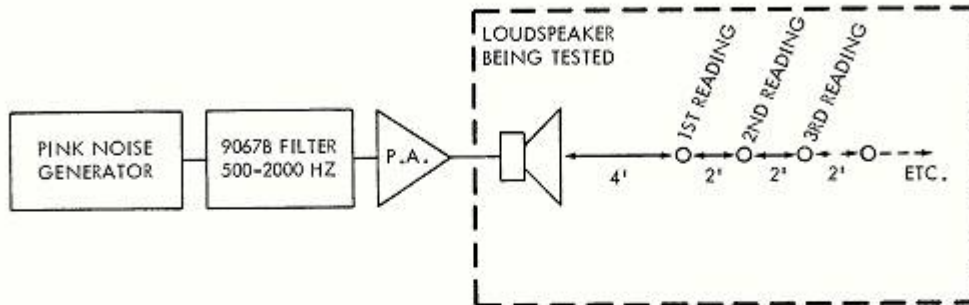


Figure 7. Measuring D_c

Then measure D_c by starting at four feet from the loudspeaker and taking a SLM reading every two feet away from it until well past D_c . Plot these readings and read D_c as shown in Figure 8.

The mean Q for this loudspeaker in the range from 500 Hz to 2000 Hz (ideal for defining the articulation of a sound system) can then be calculated with the following equation:

$$Q = \frac{(D_c)^2}{0.01988TR}$$

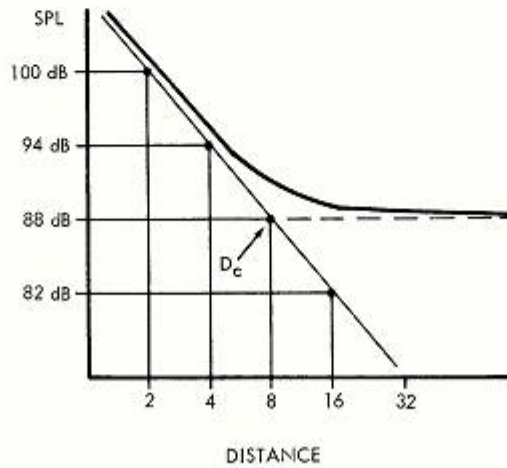


Figure 8. Plotting D_c

Calculating the Efficiency of a Loudspeaker

This same loudspeaker can be taken out in the back yard and a 1-watt reading can be measured over the same frequency range (500 - 2000 Hz) at 4', and from that on-axis sensitivity rating and its Q, its efficiency can be calculated:

$$\% \text{ efficiency} = \text{antilog}_{10} \left[\frac{4' \text{ one watt rating} - (10 \log_{10} Q + 107.47)}{10} \right] \times 100$$

SUMMARY

These simple formulas, an HP-35, your Acousto-Voicing test gear, and a small amount of time; allow you to obtain knowledge of your competitor's equipment that even they don't know. You can find out which of two loudspeakers is most efficient rather than just which has the most sensitivity. Remember, your competitors only know Q as a paragraph in your specification. Your designs prove what knowledge of Q can do for the customer.