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The Measurement of Acoustic Impedance and the Absorption Coefficient of Porous Materials

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SYNOPSIS: Various ways of determining the acoustic impedance and the absorption coefficient of porous materials from measurements on the standing waves in tubes are discussed. In all cases the material under investigation is placed at one end of the tube and the sound is introduced at the other end. Values of the coefficient of absorption of a number of commonly used damping materials as obtained by one of the methods are given. Several types of built-up structures are shown to have a greater absorption coefficient for low frequency sound waves than is conveniently obtainable by a single layer of material.

THE most commonly used method of determining the sound absorption coefficient of a material is that devised by the late Professor W. C. Sabine. In this method the reverberation time of a room is measured before and after the introduction of a definite amount of the material. This method has the great merit that the values so determined usually apply to the materials precisely as they are ordinarily used in rooms for damping purposes. However, it is tedious and requires a very quiet room and large samples of the materials. A simpler scheme has been devised by H. O. Taylor,[1] in which the absorbing material is placed at one end of a tube. The coefficient of absorption is determined from a measurement of the ratio of maximum to minimum pressures of the standing waves within the tube when sound is introduced at the open end. Thus only a small sample of the material is required and with suitable apparatus the measurements can be made with great facility. In this paper several modifications of Taylor's tube method are discussed; in addition, it is shown that by a similar method it is possible to determine not only the absorption coefficient but also the acoustic impedance, a quantity which is playing an important part in present day applied acoustics.

GENERAL THEORY

Consider a tube of length l, which is filled with a medium having a propagation constant $P = \alpha + i\beta$ and a characteristic acoustic im-

[1] *Phys. Rev.*, II, 1913, p. 270.

$P = K = \frac{\omega}{c} = \alpha + i\beta$

per $l=0$ $$p = \zeta e^{i\omega t}\left[\cdot \frac{Z_2}{Z_0} \cosh Px - \sinh Px\right] Z_0$$

per $x=0$
$$p = R\zeta_1 \cdot \frac{Z_2 \cos \beta l + iR \sin \beta l}{R \cos \beta l + i Z_2 \sin \beta l}$$

pedance[2] equal to Z_0 per unit area. At one end, O, let the velocity be uniform over the whole cross-section and equal to $\xi_1 e^{i\omega t}$. At a distance l from O let the tube be terminated by the material which is to be investigated, and the acoustic impedance of which may be rep-

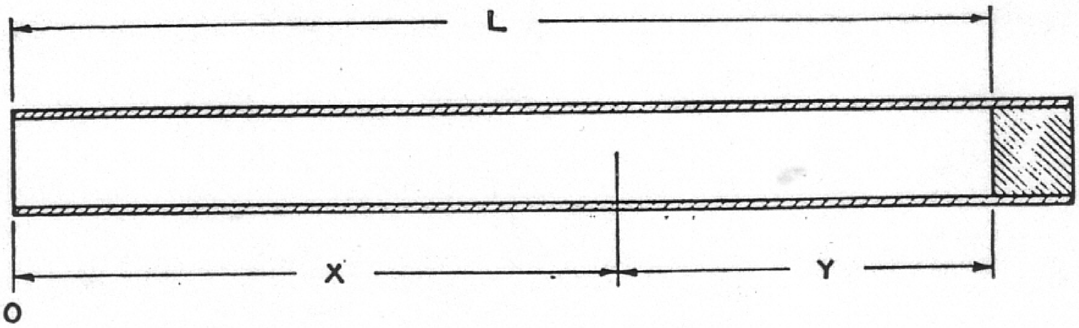

Fig. 1

resented by $Z_2 = R_2 + iX_2$ per unit area. Under these conditions, the pressure, p, at any point in the tube at a distance x from O, is by analogy with the electrical transmission line

(*)
$$p = \xi_1 e^{i\omega t} \left[\frac{Z_2 \cosh Pl + Z_0 \sinh Pl}{Z_0 \cosh Pl + Z_2 \sinh Pl} \cosh Px - \sinh Px \right] Z_0.^{3} \qquad (1)$$

If there is no attenuation along the tube, we get, on dropping the time factor,

$$p = R\xi_1 \left[\frac{Z_2 \cos \beta l + iR \sin \beta l}{R \cos \beta l + iZ_2 \sin \beta l} \cos \beta x - i \sin \beta x \right], \qquad (2)$$

where $R = c\rho$, the product of the velocity of propagation along the tube and the density of the medium, and

$$\beta = \frac{2\pi f}{c}.$$

Equation (2) indicates numerous possible ways of determining Z_2, e.g., from the values of ξ_1 and of p at any point in the tube; from the pressures for two values of either x or l, if ξ_1 is constant; from the pressures at any point in the tube for the unknown and for a known value of Z_2; from the magnitude of p as a function of either x or l. However, we shall confine our discussion to three methods, which appear to be most practicable.

[2] The term acoustic impedance as here used may be defined as the ratio of pressure to volume velocity; the characteristic impedance is this impedance if the tube were of infinite length.

[3] J. A. Fleming, "Propagation of Electric Currents in Telephone and Telegraph Conductors," page 98; 3d Ed.

(*) integro e ottengo velocità quindi calcolo $Z = \frac{P}{v}$

(a) *Pressure Measured at Two Points in the Tube*

It has already been pointed out that the impedance Z_2 can be determined if the relative phase and magnitude of the pressures at any two points in the tube are known. However, from the standpoint of convenience and precision it appears best to measure the pressures at the reflecting surface and at a point a quarter of a wave-length away. We then have at the reflecting surface $x = l$ and

$$p_2 = R\xi_1\left[\frac{R_2 + iX_2}{R\cos\beta l + iZ_2\sin\beta l}\right],$$

and for the point $x = l - \frac{\lambda}{2} = l - \frac{\pi}{2\beta}$,

$$p_1 = R\xi_1\left[\frac{iR}{R\cos\beta l + iZ_2\sin\beta l}\right],$$

so that

$$\frac{p_2}{p_1} = \frac{X_2 - iR_2}{R} \equiv Ae^{i\varphi}.$$

Hence

$$\left.\begin{aligned} R_2 &= -AR\sin\varphi, \\ X_2 &= AR\cos\varphi, \\ |Z_2| &= AR. \end{aligned}\right\} \tag{3}$$

If the coefficient of reflection is expressed as [4]

$$Ce^{i\psi} = \frac{Z_2 - R}{Z_2 + R}, \tag{4}$$

we get

$$C = \left[\frac{1 + 2A\sin\varphi + A^2}{1 - 2A\sin\varphi + A^2}\right]^{1/2},$$

where

$$\varphi = \tan^{-1}\frac{2A\cos\varphi}{A^2 + 1}. \tag{5}$$

The absorption coefficient, which is generally defined as the ratio of absorbed to incident power, is equal to $1 - |C|^2$.

(b) *Tube of Constant Length; the Absolute Value of the Pressure Measured at Points along the Tube*

The method discussed under this section is that adopted by H. O. Taylor for measuring the absorption coefficient of porous materials.

[4] I. B. Crandall, "Theory of Vibrating Systems and Sound," page 168.

For the absolute value of the pressure at any point in the tube we get from equation (2)

$$|p| = \left[\frac{R_2^2 + X_2^2 + R^2 + (R_2^2 + X_2^2 - R^2)\cos 2\beta y + 2X_2R\sin 2\beta y}{R_2^2 + X_2^2 + R^2 - (R_2^2 + X_2^2 - R^2)\cos 2\beta l - 2X_2R\sin 2\beta l}\right]^{1/2} R\xi_1, \quad (6)$$

where $y = l - x$.

$|p|$ has maximum or minimum values when

$$\tan 2\beta y = \frac{2X_2R}{X_2^2 + R_2^2 - R^2}; \quad (7)$$

for the maximum value $2\beta y$ lies in the first and for the minimum, in the third quadrant. We therefore get

$$\frac{|p|_{\max}}{|p|_{\min}} = \left[\frac{X_2^2+R_2^2+R^2+\sqrt{(X_2^2+R_2^2-R^2)^2+4X_2^2R^2}}{X_2^2+R_2^2+R^2-\sqrt{(X_2^2+R_2^2-R^2)^2+4X_2^2R^2}}\right]^{1/2} \equiv A. \quad (8)$$

Let y_1 be the value of y for which the pressure is a maximum; we then have from (7) and (8) and (4)

$$R_2 = \frac{2AR}{(A^2 + 1) - (A^2 - 1)\cos 2\beta y_1}, \quad (9)$$

$$X_2 = \frac{R(A^2 - 1)\sin 2\beta y_1}{(A^2 + 1) - (A^2 - 1)\cos 2\beta y}, \quad (10)$$

$$C_1 = \frac{A - 1}{A + 1}, \quad (11)$$

$$\Psi = 2\beta y_1.$$

The relation (11) can be derived more simply on the classical theory, as it was done by H. O. Taylor. A derivation of (11) is given by Eckhardt and Chrisler,[5] which differs from that of H. O. Taylor. From their derivation it would appear that for (11) to be valid the length of the tube should be adjusted for resonance and that the change in phase at the reflecting surface should be small. The derivation here given shows that (11) is general; it implies only that the waves be plane and that there be no dissipation of power along the tube.

[5] Scientific Paper of the Bureau of Standards, No. 526, page 56.

(*c*) *Tube of Variable Length. Pressure Measured at the Source*

The absolute value of the pressure at the driving end of the tube according to (2) is

$$|p_1| = \left[\frac{R_2^2 + X_2^2 + R^2 + (R_2^2 + X_2^2 - R^2)\cos 2\beta l + 2X_2R\sin 2\beta l}{R_2^2 + X_2^2 + R^2 + (R_2^2 + X_2^2 - R^2)\cos 2\beta l - 2X_2R\sin 2\beta l}\right]^{1/2} R\xi_1$$

and $|p_1|$ is a maximum or a minimum when

$$\tan 2\beta l = \frac{2X_2R}{R_2^2 + X_2^2 - R^2}.$$

For the maximum value $2\beta l$ lies in the first and for the minimum, in the third quadrant. We therefore have

$$\frac{|p_1|_{\max}}{|p_1|_{\min}} = \frac{X_2^2 + R_2^2 + R^2 + \sqrt{(X_2^2 + R_2^2 - R^2)^2 + 4X_2^2R^2}}{X_2^2 + R_2^2 + R^2 - \sqrt{(X_2^2 + R_2^2 - R^2)^2 + 4X_2^2R^2}} \equiv A.$$

By analogy from the equations derived in section (*b*) above, we see that

$$R_2 = \frac{2\sqrt{A}R}{(A+1) - (A-1)\cos 2\beta l_1},$$

$$X_2 = \frac{R(A-1)\sin 2\beta l_1}{(A+1) - (A-1)\cos 2\beta l_1},$$

$$C = \frac{\sqrt{A} - 1}{\sqrt{A} + 1},$$

$$\Psi = 2\beta l_1,$$

where l_1 is the length of the tube when p_1 has a maximum value.

Discussion of the Precision of the Methods

Of the three methods of measuring impedance discussed above, the first is undoubtedly the simplest and most convenient, if an a.c. potentiometer is available. Theoretically, in this case the impedance may be determined with a high degree of precision. However, the method presupposes that the points where the pressures are measured are exactly a quarter of a wave-length apart; a more detailed analysis shows that, if A is small, variations in this distance will have a large effect on both the ratio of the pressures and their phase difference. It therefore is necessary to keep the temperature of the tube accurately constant or else to determine the distance corresponding to a quarter of a wave-length before each measurement. A precise determination of the point a quarter of a wave-length from the reflecting surface may be made by placing a smooth metal block at the reflecting end and finding then the position in the tube at which the pressure is a minimum.

In the other two methods it is relatively less important that the temperature be maintained constant, for the ratio of pressures is affected very little by any temperature variations. In the third method, where the length of the tube is varied, the expressions for R_2 and X_2 are the same as in (*b*), except that in place of the ratio of pressures they involve the square root of this ratio. For small values of pressure ratios the precision is therefore somewhat greater. However, for high values of reflection the ratio becomes very large and great care is required in the experimental set up to prevent errors creeping into the measurements through extraneous vibrations and stray electromotive forces in the measuring circuit. The main advantage of the method in which the pressure at the source only is measured is that a short length of exploring tube is required. If measurements down to a frequency of 60 cycles are made, the tube length must be at least 8 feet. An exploring tube reaching the whole length would ordinarily introduce too much attenuation if it were of sufficiently small bore to prevent resonance effects at the lower frequencies.

Experimental Procedure

In the case of the experimental results here reported the measurements were all made by the method outlined in section (*c*), i.e., the pressures were measured at the source while the length of the tube was varied. The experimental set up is shown in Fig. 2. A piece of Shelby

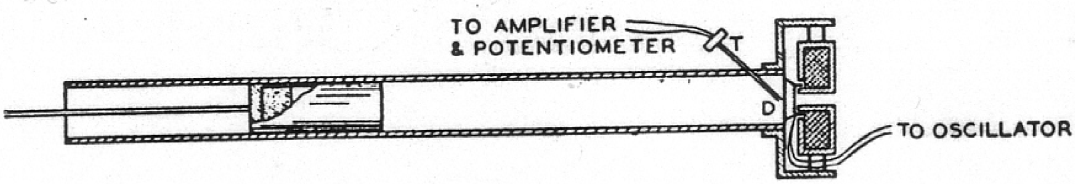

Fig. 2—Diagram of apparatus

steel tubing, 9 feet long, of 3″ internal diameter, and with 1/4″ wall, was fitted with a piston carrying the absorbing material. This piston was made up of a brass tube one foot long with a wall 1/64″ thick, the far end of which was closed with a one-inch brass block. To insure the propagation of plane waves and a constant velocity at the source, the diaphragm at *D* had a diameter of 2⅞″, and a mass of about 100 grams. This was driven with a coil 2″ in diameter situated in a radial magnetic field. The annular gap between the edge of the diaphragm and the interior of the tube was closed by a flexible piece of leather. To prevent vibrations of the magnet from getting to the tube, the magnet was held in position by flexible supports. The exploring tube t was about 5″ long with a 1/16″ bore which led to the transmitter, T. The voltages generated by the transmitter were measured with an amplifier and an a.c. potentiometer. The potentiometer was used because with it small voltages can be measured and errors due to harmonics are avoided. The proper functioning of the apparatus was determined by measuring the coefficient of reflection with no absorbing material in the piston. Theoretically the reflection should then be practically 100 per cent. The pressure ratios that were actually observed were of the order of 12,000 which corresponds to a reflection coefficient of 98 per cent. Evidently some extraneous pressures or voltages were still present. However, no attempt was made to reduce these further as the materials tested had a reflection coefficient considerably less than this value.

Experimental Results

A brief study was made of the absorption of hair felt, as there is an appreciable variation in the data given by various investigators on the absorption frequency characteristic of felts of presumably the same

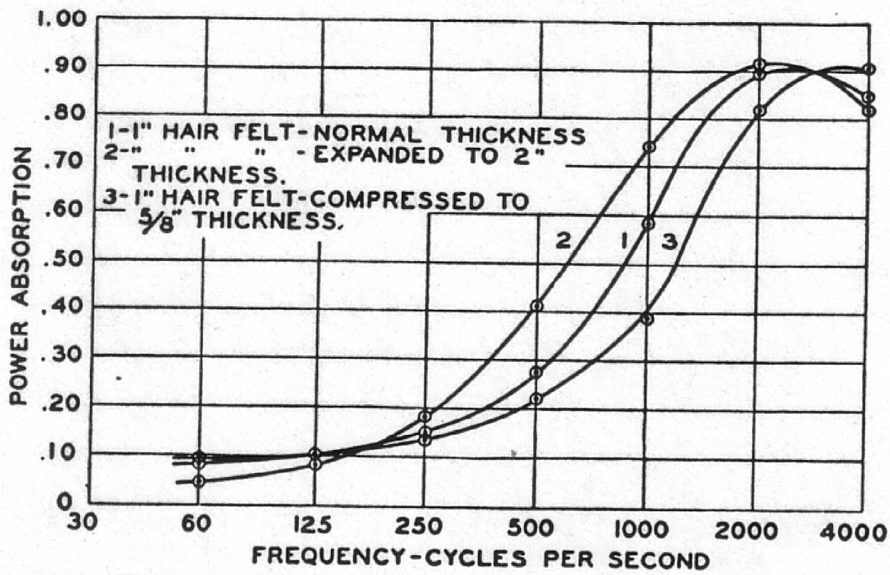

Fig. 3—Power absorbed by hair felt

type. After measurements on several samples it was evident that concordant results could not be expected as the absorption varied considerably with the packing of the felt. This point is illustrated by the curves shown in Fig. 3. These curves were all obtained on the same piece of hair felt but with different degrees of packing. It is thus evident that a felt which has become loosened by handling may have an absorption frequency characteristic quite unlike that of a new piece.

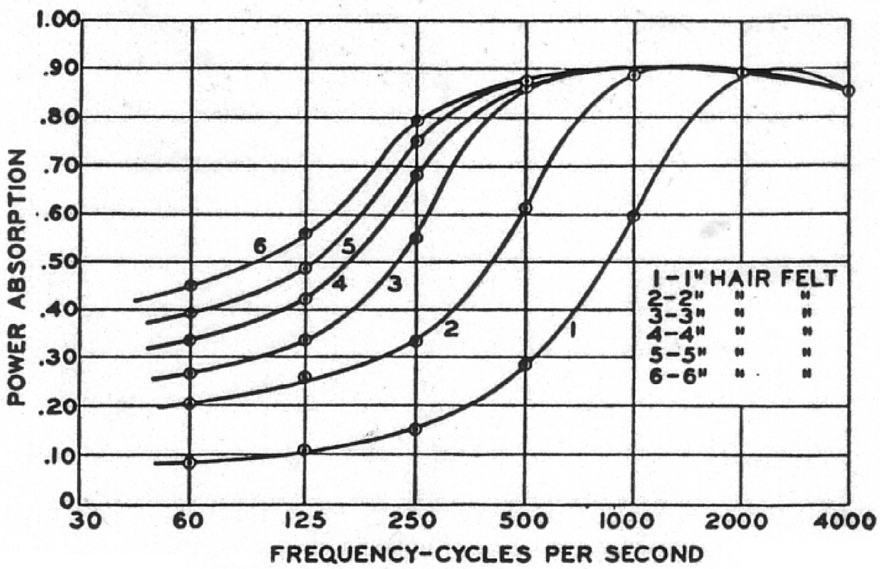

Fig. 4—Power absorbed by hair felt

In Fig. 4 are given the absorption coefficients for various thicknesses of hair felt. These values are in general agreement with those obtained by the reverberation method according to published results. Exact agreement is not to be expected, for the values here given apply only to sound waves having a perpendicular incidence on materials solidly backed by a hard surface. When the materials are applied in a room, the support is often more flexible and the absorption is partly due to inelastic bending. However, the agreement between the sets of values is sufficiently good to show that the results obtained by the simpler tube method may be used to get a good approximation to the values of absorption of the materials when applied in rooms for damping purposes.

Measurements have been made on a large number of porous materials. Although most of these materials are very good absorbers at the higher frequencies, none of them were found to be very efficient in the lower frequency region. Uniform absorption over most of the frequency range was found only in materials which are relatively inefficient absorbers. High absorption at the lower frequencies was obtained only when the thickness of the material was greatly increased. This fact is typically illustrated by the curves of absorption for hair felt given in Fig. 4.

When a sound wave of low frequency is reflected from a wall covered

[*Editor's Note:* Page 9, which consists entirely of Table I, absorption coefficients of various sandwich structures, has been omitted.]

with a porous material, the velocity of the air particles near the reflecting surface is small and hence there can be but little absorption. We may look at the phenomenon of reflection in still another way. In order to have a small coefficient of reflection the mechanical impedance of the wall per unit area should, as nearly as possible, be equal to the acoustic impedance of the air per unit area. The reason for the high reflection at low frequencies by a rigid wall covered with a porous material lies in its high stiffness reactance. At a given frequency this reactance can be compensated by loading the air near the reflecting surface. This may be accomplished in various ways. One of these ways is to place at a short distance from the wall a second wall which is porous or perforated. This arrangement has the effect of covering the wall with a multiplicity of resonators, which may be given any desired resonance frequency by properly proportioning the size, length and number of perforations and the spacing of the walls. The surface of the walls forming the air space should be absorbing or else the space should be provided with absorbing material.

To get a wider absorption band two or more perforated walls with proper spacing may be used, as this arrangement is equivalent to an aggregate of multiple resonators. The values of absorption coefficients of a number of structures of this type are given in the accompanying table. The measurements refer to sound which is incident from right to left as the structures are given in the table. The building board referred to in the table is a commercial type of insulating-board one inch thick with 400 1/4 inch by 3/4 inch holes per square foot. The felt in all cases is one-inch hair felt. These values show that relatively high absorption may be obtained at low as well as at high frequencies without an excessive amount of absorbing material. The use of combinations of absorbing materials, such as are given in the table, offers the advantage that more uniform damping at all frequencies can be obtained, and the degree of damping can be readily controlled by covering the proper area of surface. These two factors have become increasingly important in studio and auditorium design, with improved technique in recording and reproducing speech and music.

EDITOR'S NOTES

Page 2: Footnote 2 is confusing. Today's definition of the characteristic acoustic impedance of a medium is not $p/\dot{V}$ (volume velocity), which equals $\rho c/S$ or $Z_{acoustic}$ but $p/\dot{\xi}$ (linear velocity), which equals ρc. Hence, when Wente says Z_0 per unit area, which requires division by area S giving $\rho c/S^2$, he means Z'_{ac} *for* unit area, which implies multiplication by area S. Then $p/\dot{V}$ times S will equal the desired ρc or traditional Z_0.

Page 2: An understanding of Equations 1 and 2 will be helped by J. A. Fleming's Equation 67, *Propagation of Electric Currents in Telephone and Telegraph Conductors*, 3rd. ed., Constable & Son, London, 1927; also by P. M. Morse's *Vibration and Sound*, 2nd ed., McGraw-Hill, New York, 1948, pp. 133–143. Morse is more difficult than Fleming, but almost essential for the later papers in this portion of the volume.

Page 3: Note that in Equation 3, A is $|p_2|/|p_1|$.

Page 5: In method (c), since $\beta \ell = \omega \ell / c$, we can vary either ℓ or ω and get the same result. Thus when Wente varies ℓ, he gets a [tan $\beta \ell$] function, or a [–cot $\beta \ell$] function, and so on. Flanders, in paper 39, keeps ℓ fixed but varies ω and gets the same curve.

Page 6: Figure 2 shows that Wente is clearly measuring the pressure component of the standing-wave system, unlike Taylor or Paris.

Page 8, Middle of the page: Note that the absorption coefficient for oblique incidence has still not been solved.

Page 10, line 8: Wente's comments on "high stiffness reactance" also apply to the insensitivity of Kennelly and Kurokawa's instrument when used away from resonance (see Paper 41).